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ABSTRACT

This report deals with scattering of electromagnetic waves from an infinite cylinder of uniform but complex dielectric constant. Its main purpose is to make available a set of curves showing the dependence of transmission and reflection coefficients and their phases on plasma frequency and cylinder radius. Two different collision frequencies are used. All quantities have been normalised to eliminate the dependence on the operating frequency and the distance between the transmitting and receiving horns. The incident wave is assumed to be a plane wave traveling in a direction normal to the cylinder axis. Both the transverse magnetic case and the transverse electric case are considered.

TISIA

ACKNOWLEDGEMENTS

The authors wish to thank the Data Analysis Section of Group 312 and the Lincoln Laboratory Drafting Room for the careful work done in preparing the graphs.

SCATTERING FROM A HOMOGENEOUS PLASMA CYLINDER OF INFINITE LENGTH

by

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and

L. G. Peterson

The solution of a scattering problem involves three steps

1) Determination of the electric field $\underline{E}$ from the equation

$$\nabla^2 \underline{E} + k_o^2 \epsilon' \underline{E} = \nabla(\nabla.\underline{E}) \tag{1}$$

or of the magnetic field $\underline{H}$ from the equation

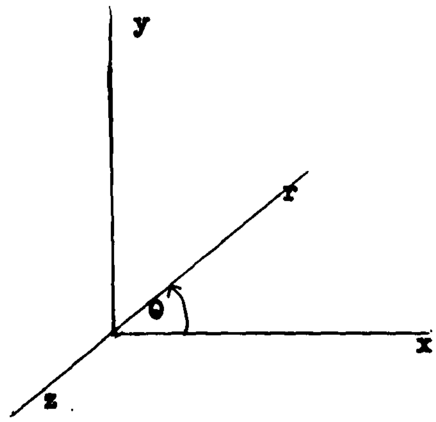
$$\nabla^2 \underline{H} + k_o^2 \epsilon \underline{H} = (\nabla \underline{H}) \times \frac{\nabla\epsilon'}{\epsilon'} \tag{2}$$

where $k_o = \omega \sqrt{\epsilon_o \mu_o}$ is the free space propagation constant and ϵ' is the complex dielectric constant*. Which of the two equations 1 or 2 is solved depends on which is simpler for a given geometry and incident radiation. Only one equation has to be solved because once $\underline{E}$ has been found $\underline{H}$ can be determined with the help of Maxwell's equations and similarly once $\underline{H}$ has been found $\underline{E}$ can be determined.

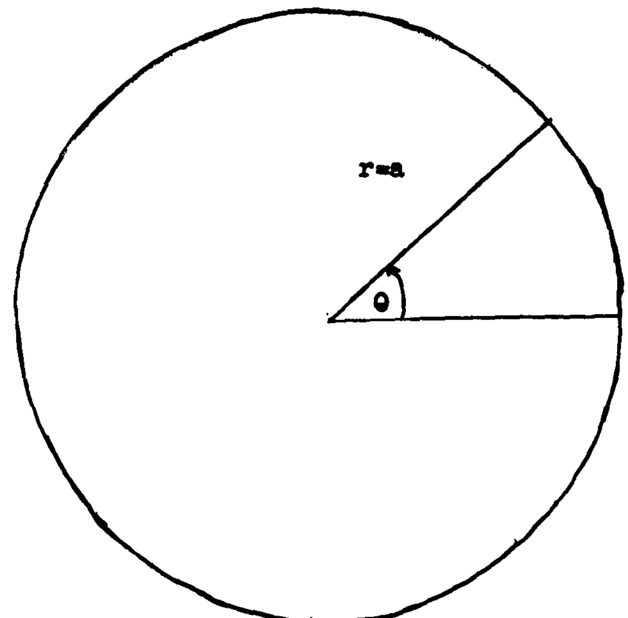
2) Satisfaction of the radiation condition at infinity.

3) Satisfaction of the boundary conditions at the interface between regions consisting of different media.

* See appendix 1



Coordinate System



Cross Section of plasma cylinder
with axis coinciding with the z-axis

Outside the cylinder $\epsilon' = 1$ and inside the cylinder ϵ' is taken to be a constant given by

$$\epsilon' = 1 - \frac{\bar{f}_p^{\,2}\,(1-i\bar{\nu})}{1+\bar{\nu}^2}$$

where $\bar{f}_p = \frac{f_p}{f}$ is the ratio of the plasma frequency f_p to f the frequency of the incident wave,

and $\bar{\nu} = \frac{\nu}{2\pi f}$ is the ratio of ν the electron collision frequency to $2\pi f$ the circular frequency of the incident wave.

For the transverse magnetic case the incident electric field is parallel to the z-axis. In this case equation 1 is simpler than equation 2 and reduces to the scalar equation

$$\nabla^2 E_z + k_o^{\,2}\,\epsilon'\,E_z = 0$$

Neglecting the time depedence $e^{-i\omega t}$ the incident field is given by

$$\underline{E}^i = \underline{i}_z E_z^i = \underline{i}_z E_o e^{i 2\pi\bar{r}\cos\theta} = \underline{i}_z E_o \sum_{n=o}^{\infty} \epsilon_n i^n J_n(2\pi\bar{r}) \cos n\theta$$

where $\underline{i}_z$ is a unit vector in the z direction

$$\bar{r} = \frac{r}{\lambda_o}$$

λ_o the free space wave length of the incident radiation

$$\epsilon_n = \begin{cases} 1 & \text{when } n = o \\ 2 & \phantom{\text{when }} n \geq 1 \end{cases}$$

Let the scattered field outside the cylinder be represented by

$$\underline{E}^s = \underline{i}_z E_z^s = \underline{i}_z E_o \sum_{n=o}^{\infty} A_n \epsilon_n i^n H_n^{(1)} (2\pi\bar{r}) \cos n\theta$$

and the total field inside the cylinder by

$$\underline{E}^t = \underline{i}_z E_z^t = \underline{i}_z E_o \sum_{n=o}^{\infty} C_n \epsilon_n i^n J_n(2\pi\bar{r}\sqrt{\epsilon'}) \cos n\theta$$

The choice of the Bessel function in E_z^t insures that the solution will be finite at the origin and that of the Hankel function of the first kind in E_z^s insures that the scattered field at large distances from the cylinder axis will behave as an outgoing cylindrical wave and the radiation condition will be satisfied.

The constants A_n and C_n can be determined from the boundary conditions. To obtain reflection and transmission coefficients and their

phases the scattered field outside the cylinder must be known and hence A_n must be found. Using Maxwell's equation we find

$$H_\theta^{\,i} = \frac{iE_o}{\eta_o} \sum_{n=o}^{\infty} \epsilon_n i^n \frac{\partial J_n(2\pi\bar{r})}{\partial(2\pi\bar{r})} \cos n\theta$$

$$H_\theta^{\,s} = \frac{iE_o}{\eta_o} \sum_{n=o}^{\infty} A_n \epsilon_n i^n \partial \frac{H_n^{(1)}(2\pi\bar{r})}{\partial (2\pi\bar{r})} \cos n\theta$$

$$H_\theta^{\,t} = \frac{iE_o\sqrt{\epsilon'}}{\eta_o} \sum_{n=o}^{\infty} C_n \epsilon_n i^n \partial \frac{J_n(2\pi\bar{r}\sqrt{\epsilon'})}{\partial (2\pi\bar{r}\sqrt{\epsilon'})} \cos n\theta$$

where $\eta_o = \sqrt{\frac{\mu_o}{\epsilon_o}}$ is the impedance of free space.

The boundary conditions at r=a require that

$$E_z^s + E_z^i = E_z^t$$

$$H_\theta^s + H_\theta^i = H_\theta^t$$

Substituting the series expressions for the field components and letting $\bar{a} = \frac{a}{\lambda_o}$ we obtain

$$A_n H_n^{(1)}(2\pi\bar{a}) + J_n(2\pi\bar{a}) = C_n J_n(2\pi\bar{a}\sqrt{\epsilon'})$$

$$A_n \frac{\partial H_n^{(1)}(2\pi\bar{a})}{\partial(2\pi\bar{a})} + \frac{\partial J_n(2\pi\bar{a})}{\partial(2\pi\bar{a})} = C_N \sqrt{\epsilon'} \frac{\partial J_n(2\pi\bar{a}\sqrt{\epsilon'})}{\partial(2\pi\bar{a}\sqrt{\epsilon'})}$$

From these using the expression for the derivative of a Bessel function

$$A_n = A_{n1} + iA_{n2} = \frac{-1}{1 + ia_n}$$

where

$$a_n = \frac{\sqrt{\epsilon'}\, J_{n+1}(2\pi\bar{a}\sqrt{\epsilon'})\, N_n(2\pi\bar{a}) - J_n(2\pi\bar{a}\sqrt{\epsilon'})\, N_{n+1}(2\pi\bar{a})}{\sqrt{\epsilon'}\, J_{n+1}(2\pi\bar{a}\sqrt{\epsilon'})\, J_n(2\pi\bar{a}) - J_n(2\pi\bar{a}\sqrt{\epsilon'})\, J_{n+1}(2\pi\bar{a})}$$

Similarly for the transverse electric case the incident magnetic field is parallel to the z-axis, and neglecting time dependence is given by

$$\underline{H}^i = \underline{i}_z H_z^i = \underline{i}_z \frac{E_o}{\eta_o} e^{i2\pi\bar{r}\cos\theta} = \underline{i}_z \frac{E_o}{\eta_o} \sum_{n=o}^{\infty} \epsilon_n i^n J_n(2\pi\bar{r}) \cos n\theta$$

proceeding as before let the scattered magnetic field outside the cylinder be given by

$$H_z^s = \frac{E_o}{\eta_o} \sum_{n=o}^{\infty} B_n \epsilon_n i^n H_n^{(1)} (2\pi\bar{r}) \cos n\theta$$

and the total magnetic field inside the cylinder by

$$H_z^t = \frac{E_o}{\eta_o} \sqrt{\epsilon'} \sum_{n=o}^{\infty} D_n \epsilon_n i^n J_n(2\pi\bar{r}\sqrt{\epsilon'}) \cos n\theta$$

using Maxwell's equations we find

$$E_\Theta{}^i = -iE_o \sum_{n=o}^{\infty} \epsilon_n i^n \frac{\partial J_n(2\pi\bar{r})}{\partial(2\pi\bar{r})} \cos n\Theta$$

$$E_\Theta{}^s = -iE_o \sum_{n=o}^{\infty} B_n \epsilon_n i^n \frac{\partial H_n{}^{(1)}(2\pi\bar{r})}{\partial(2\pi\bar{r})} \cos n\Theta$$

$$E_\Theta{}^t = -iE_o \sum_{n=o}^{\infty} D_n \epsilon_n i^n \frac{\partial J_n(2\pi\bar{r}\sqrt{\epsilon'})}{\partial(2\pi\bar{r}\sqrt{\epsilon'})} \cos n\Theta$$

For the transverse electric case the boundary conditions which must be satisfied at r=a give

$$\frac{\partial J_n(2\pi\bar{a})}{\partial(2\pi\bar{a})} + B_n \frac{\partial H_n{}^{(1)}(2\pi\bar{a})}{\partial(2\pi\bar{a})} = D_n \frac{\partial J_n(2\pi\bar{a}\sqrt{\epsilon'})}{\partial(2\pi\bar{a}\sqrt{\epsilon'})}$$

$$J_n(2\pi\bar{a}) + B_n H_n{}^{(1)}(2\pi\bar{a}) = \sqrt{\epsilon'}\, D_n J_n(2\pi\bar{a}\sqrt{\epsilon'})$$

From which

$$B_n = B_{n1} + iB_{n2} = \frac{-1}{1 + ib_n}$$

where

$$b_n = \frac{J_{n-1}(2\pi\bar{a}\sqrt{\epsilon'})N_n(2\pi\bar{a}) - \sqrt{\epsilon'}J_n(2\pi\bar{a}\sqrt{\epsilon'})N_{n-1}(2\pi\bar{a}) + \frac{n(\epsilon'-1)}{2\pi\bar{a}\sqrt{\epsilon'}} J_n(2\pi\bar{a}\sqrt{\epsilon'})N_n(2\pi\bar{a})}{J_{n-1}(2\pi\bar{a}\sqrt{\epsilon'})J_n(2\pi\bar{a}) - \sqrt{\epsilon'}J_n(2\pi\bar{a}\sqrt{\epsilon'})J_{n-1}(2\pi\bar{a}) + \frac{n(\epsilon'-1)}{2\pi\bar{a}\sqrt{\epsilon'}} J_n(2\pi\bar{a}\sqrt{\epsilon'})J_n(2\pi\bar{a})}$$

The program makes use of an IBM library subroutine to calculate the Bessel and Neumann functions for real arguments and of the series expression together with the recursion formula for the Bessel functions to determine the Bessel functions for complex arguments. Care has been taken to avoid overflow and round off troubles in the calculation of A_n and B_n.

Let

$$\left.\begin{matrix}\vec{\underline{E}}^{s}_{M} \\ \\ \overleftarrow{\underline{E}}^{s}_{M}\end{matrix}\right\} = \begin{matrix}\text{the scattered electric field for the transverse magnetic case} \\ \\ \text{at a large distance from the axis of the cylinder}\end{matrix} \left\{\begin{matrix}\theta = 0 \\ \\ \theta = \pi\end{matrix}\right.$$

$$\left.\begin{matrix}\vec{\underline{H}}^{s}_{E} \\ \\ \overleftarrow{\underline{H}}^{s}_{E}\end{matrix}\right\} = \begin{matrix}\text{the scattered magnetic field for the transverse electric case} \\ \\ \text{at a large distance from the axis of the cylinder}\end{matrix} \left\{\begin{matrix}\theta = 0 \\ \\ \theta = \pi\end{matrix}\right.$$

$$\left.\begin{matrix}\vec{\underline{E}}^{i} \\ \\ \overleftarrow{\underline{E}}^{i}\end{matrix}\right\} = \begin{matrix}\text{the incident electric field at a large} \\ \\ \text{distance from the axis of the cylinder}\end{matrix} \left\{\begin{matrix}\theta = 0 \\ \\ \theta = \pi\end{matrix}\right.$$

$$\left.\begin{matrix}\vec{\underline{H}}^{i} \\ \\ \overleftarrow{\underline{H}}^{i}\end{matrix}\right\} = \begin{matrix}\text{the incident magnetic field at a large} \\ \\ \text{distance from the axis of the cylinder}\end{matrix} \left\{\begin{matrix}\theta = 0 \\ \\ \theta = \pi\end{matrix}\right.$$

We define the following eight quantities

$$TM = 2\pi^2\bar{r}\left|\frac{\vec{\underline{E}}^{s}_{M}}{\vec{\underline{E}}^{i}}\right|^2 = \left\{\sum_{n=0}^{\infty} \epsilon_n(A_{n1}+A_{n2})\right\}^2 + \left\{\sum_{n=0}^{\infty} \epsilon_n(A_{n2}-A_{n1})\right\}^2$$

$$\mathrm{PTM} = \text{phase of}\left(\frac{\overrightarrow{\underline{E}}^{s}_{M}}{\overrightarrow{\underline{E}}^{i}}\right) = \tan^{-1}\frac{\sum\limits_{n=o}^{\infty} \epsilon_n(A_{n2} - A_{n1})}{\sum\limits_{n=o}^{\infty} \epsilon_n(A_{n1} + A_{n2})}$$

$$\mathrm{RM} = \pi^2 \bar{r}\left|\frac{\overleftarrow{\underline{E}}^{s}_{M}}{\overleftarrow{\underline{E}}^{i}}\right|^2 = \left\{\sum_{n=o}^{\infty} \epsilon_n(-1)^n A_{n1}\right\}^2 + \left\{\sum_{n=o}^{\infty} \epsilon_n(-1)^n A_{n2}\right\}^2$$

$$\mathrm{PRM} = \text{phase of}\left(\frac{\overleftarrow{\underline{E}}^{s}_{M}}{\overleftarrow{\underline{E}}^{i}}\right) + \pi/4 - 4\pi\bar{r} = \tan^{-1}\frac{\sum\limits_{n=o}^{\infty} \epsilon_n(-1)^n A_{n2}}{\sum\limits_{n=o}^{\infty} \epsilon_n(-1)^n A_{n1}}$$

$$\mathrm{TE} = 2\pi^2 \bar{r}\left|\frac{\overrightarrow{\underline{H}}^{s}_{E}}{\overrightarrow{\underline{H}}^{i}}\right|^2 = \left\{\sum_{n=o}^{\infty} \epsilon_n(B_{n1} + B_{n2})\right\}^2 + \left\{\sum_{n=o}^{\infty} \epsilon_n(B_{n2} - B_{n1})\right\}^2$$

$$\mathrm{PTE} = \text{phase of}\left(\frac{\overrightarrow{\underline{H}}^{s}_{E}}{\overrightarrow{\underline{H}}_{i}}\right) = \tan^{-1}\frac{\sum\limits_{n=o}^{\infty} \epsilon_n(B_{n2} - B_{n1})}{\sum\limits_{n=o}^{\infty} \epsilon_n(B_{n1} + B_{n2})}$$

$$\mathrm{RE} = \pi^2 \bar{r}\left|\frac{\overleftarrow{\underline{H}}^{s}_{E}}{\overleftarrow{\underline{H}}^{i}}\right|^2 = \left\{\sum_{n=o}^{\infty} \epsilon_n(-1)^n B_{n1}\right\}^2 + \left\{\sum_{n=o}^{\infty} \epsilon_{n}(-1)^n B_{n2}\right\}^2$$

$$\mathrm{PRE} = \text{phase of} \left(\frac{\underline{\overleftarrow{H}}^{s}_{E}}{\underline{\overleftarrow{H}}^{i}} \right) + \pi/4 - 4\kappa\bar{r} = \tan^{-1} \frac{\sum_{n=0}^{\infty} \epsilon_n (-1)^n B_{n2}}{\sum_{n=0}^{\infty} \epsilon_n (-1)^n B_{n1}}$$

these are the transmission and reflection coefficients and phases appearing in the graphs.

APPENDIX I

I. Dielectric Constant

Assuming a time dependence $e^{-i\omega t}$ Maxwell's equations are

$$\nabla \times \underline{E} = i\omega\mu_o \underline{H}$$

$$\nabla \times \underline{H} = -i\omega\epsilon_o \underline{E} + \underline{J}$$

where ϵ_o and μ_o are the dielectric constant and permeability of free space respectively. For a single electron

$$m\,\ddot{\underline{x}} = e\,\underline{E} - m\nu\,\dot{\underline{x}}$$

ν being the electron collision frequency and m and e the mass and charge of the electron.

Hence

$$\dot{\underline{x}} = \frac{ie\underline{E}\,(1-i\bar{\nu})}{m\omega(1+\bar{\nu}^2)} \quad ; \quad \bar{\nu} = \frac{\nu}{\omega}$$

If $\underline{J} = Ne\,\dot{\underline{x}}$, where N is the electron density, then

$$\nabla \times \underline{H} = -i\omega\epsilon_o \underline{E} \left\{ 1 - \frac{Ne^2(1-i\bar{\nu})}{m\epsilon_o\omega^2\,(1+\bar{\nu}^2)} \right\}$$

or

$$\nabla \times \underline{H} = -i\omega\epsilon_o\epsilon'\,\underline{E}$$

where

$$\epsilon' = 1 - \frac{Ne^2\,(1-i\bar{\nu})}{m\epsilon_o\omega^2(1+\bar{\nu}^2)}$$

This is the effective complex dielectric constant.

The quantity $f_p = \sqrt{\frac{Ne^2}{4\pi^2 \epsilon_o m}}$ is usually referred to as the plasma frequency of the medium.

If $\bar{f}_p = \frac{f_p}{f}$, where f is the frequency of the incident radiation then

$$\epsilon' = 1 - \frac{\bar{f}_p^{\,2} (1-i\bar{\nu})}{1+\bar{\nu}^2}$$

This is the form in which the dielectric constant is used in the program. $\bar{f}_p^{\,2}$ and $\bar{\nu}$ are input values and ϵ' is calculated.

$\bar{\nu} = 1$

Figures 1 - 24

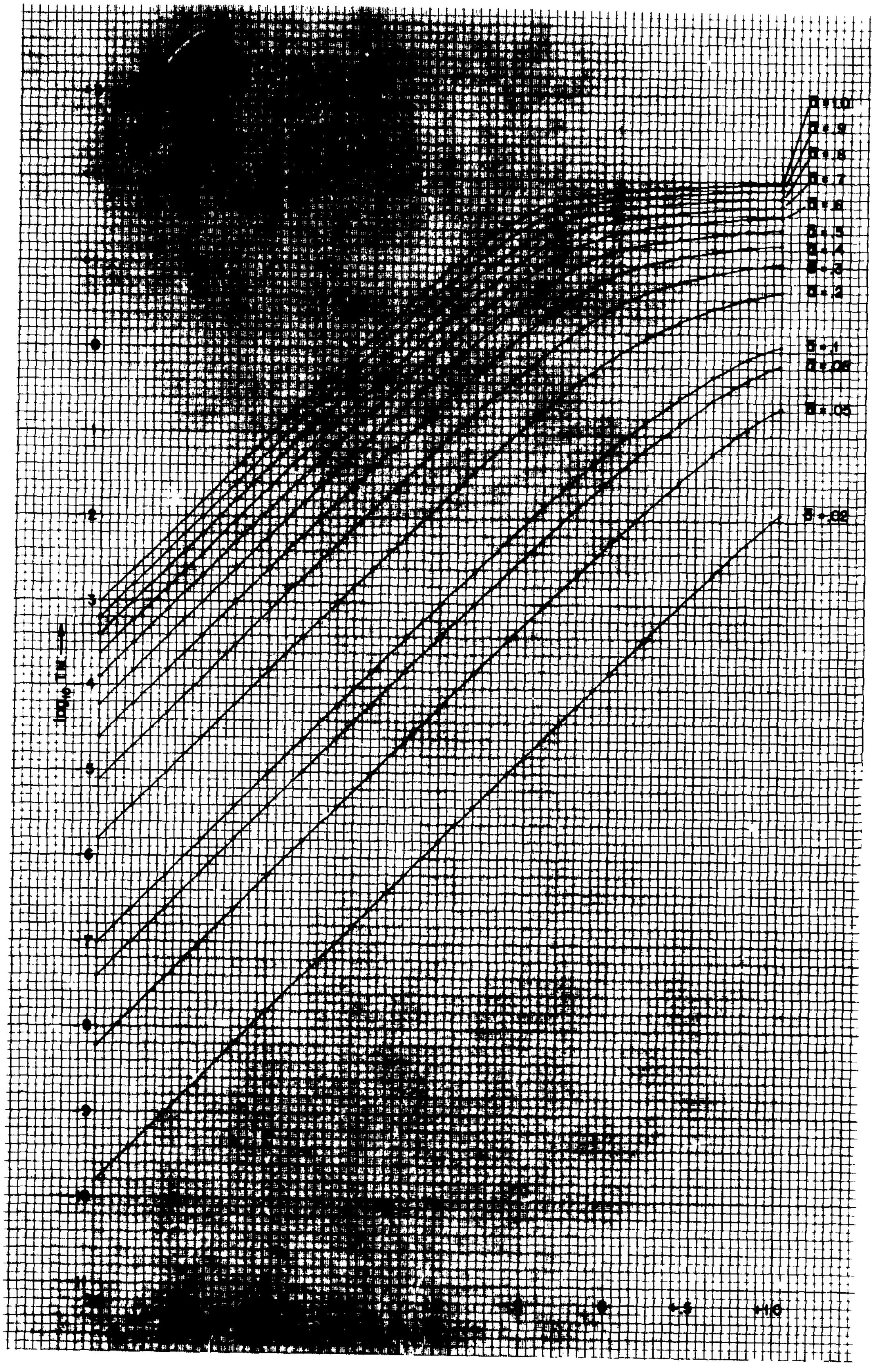


Figure 1

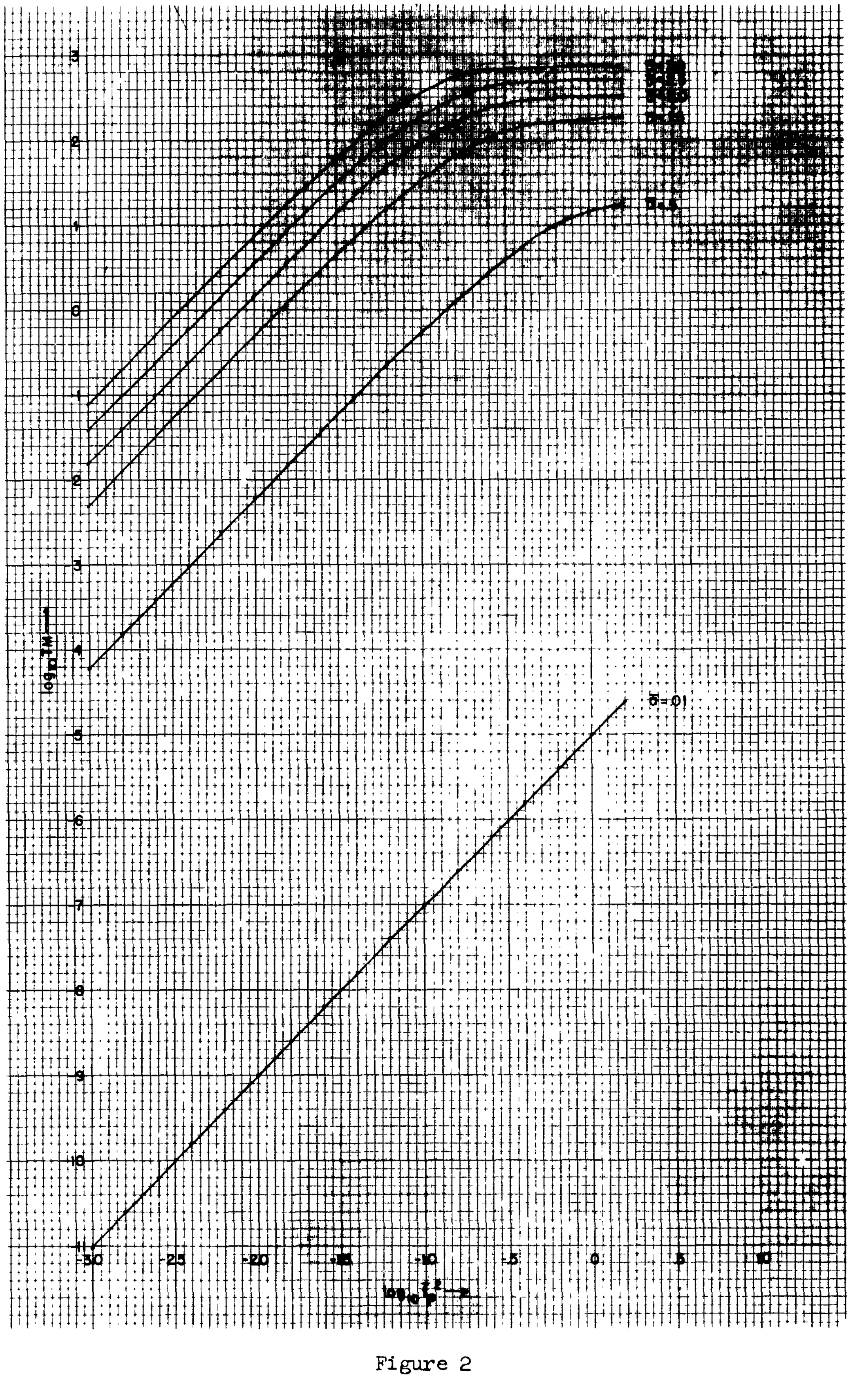


Figure 2

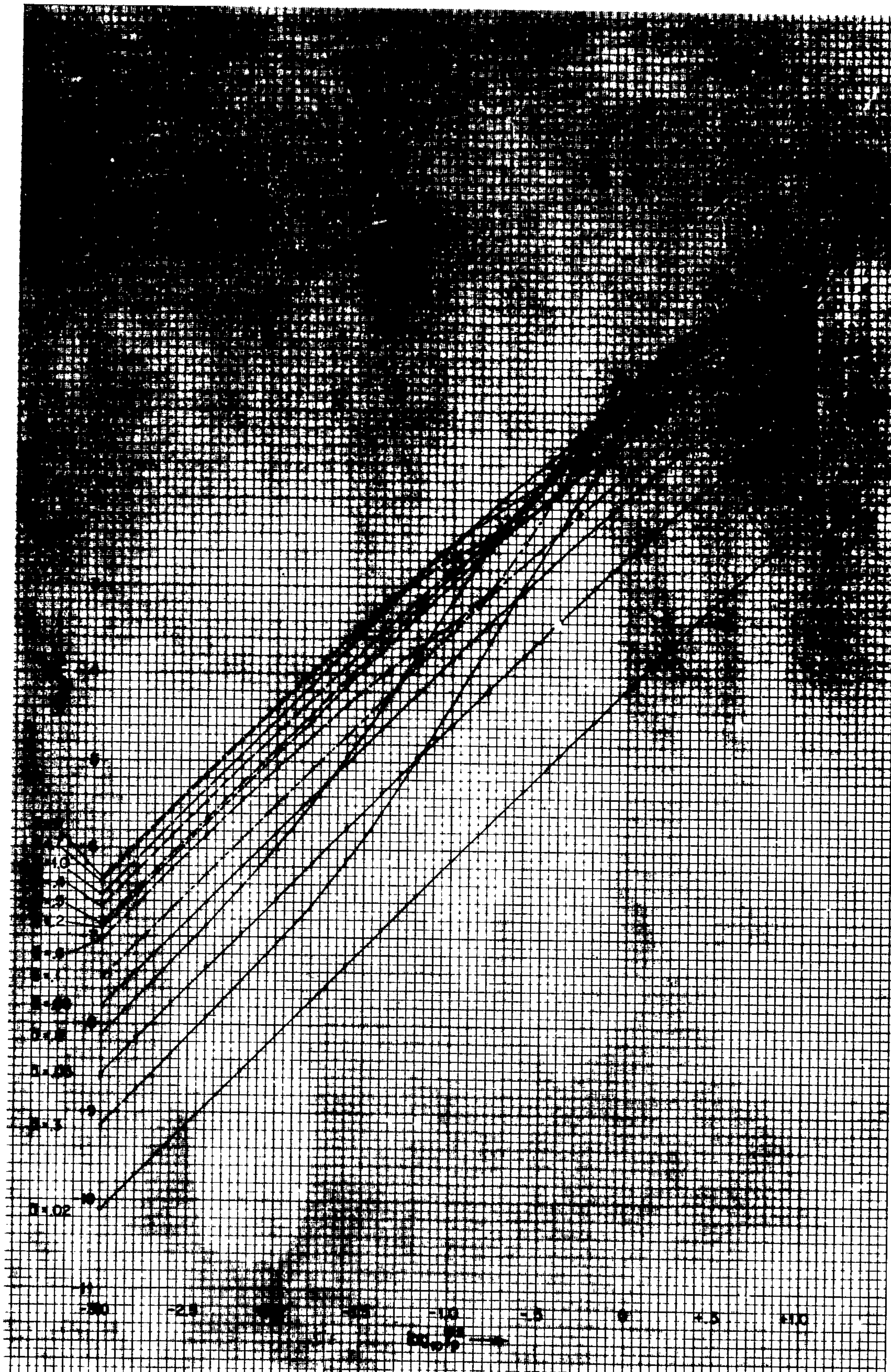
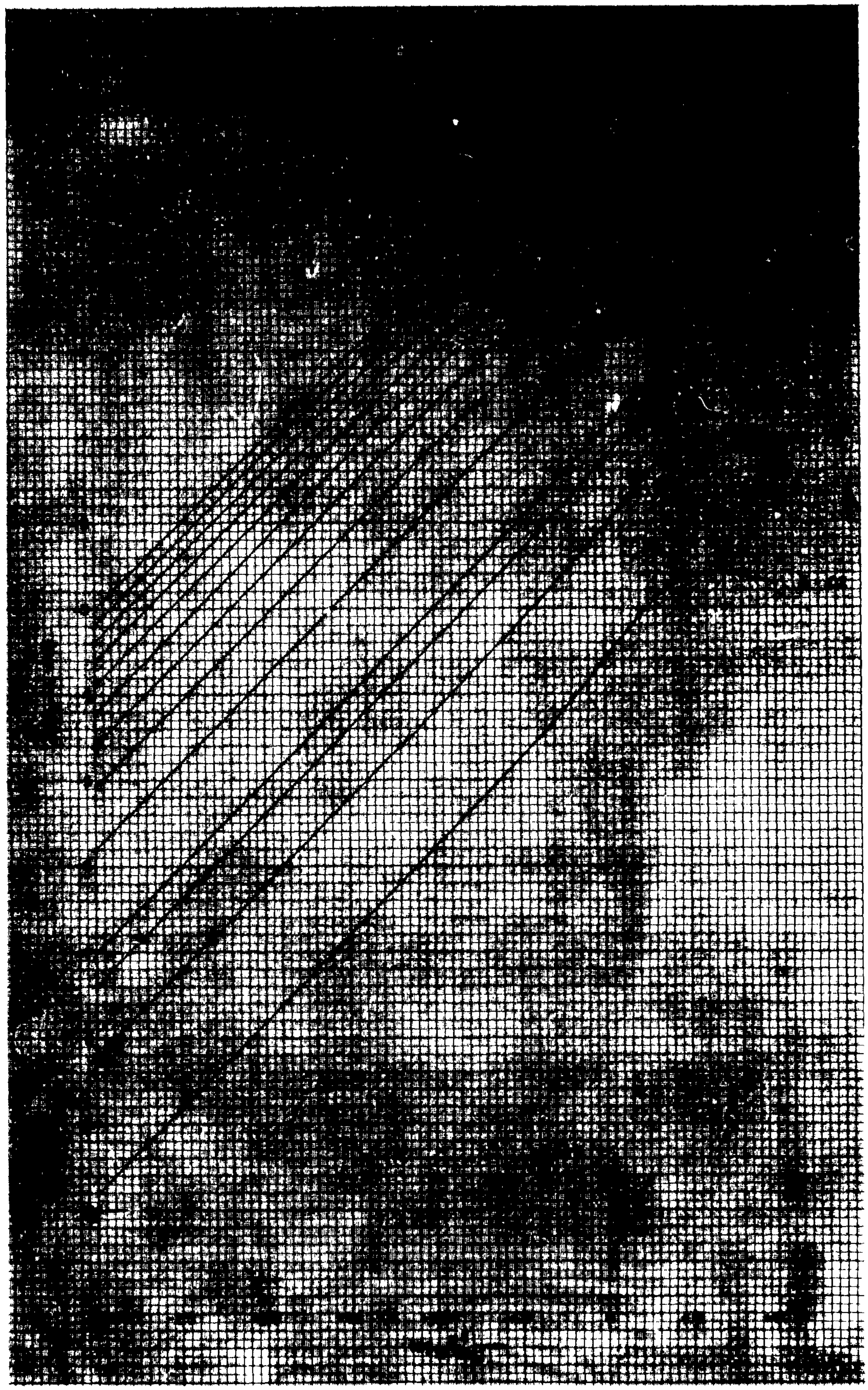


Figure 3

Figure 4



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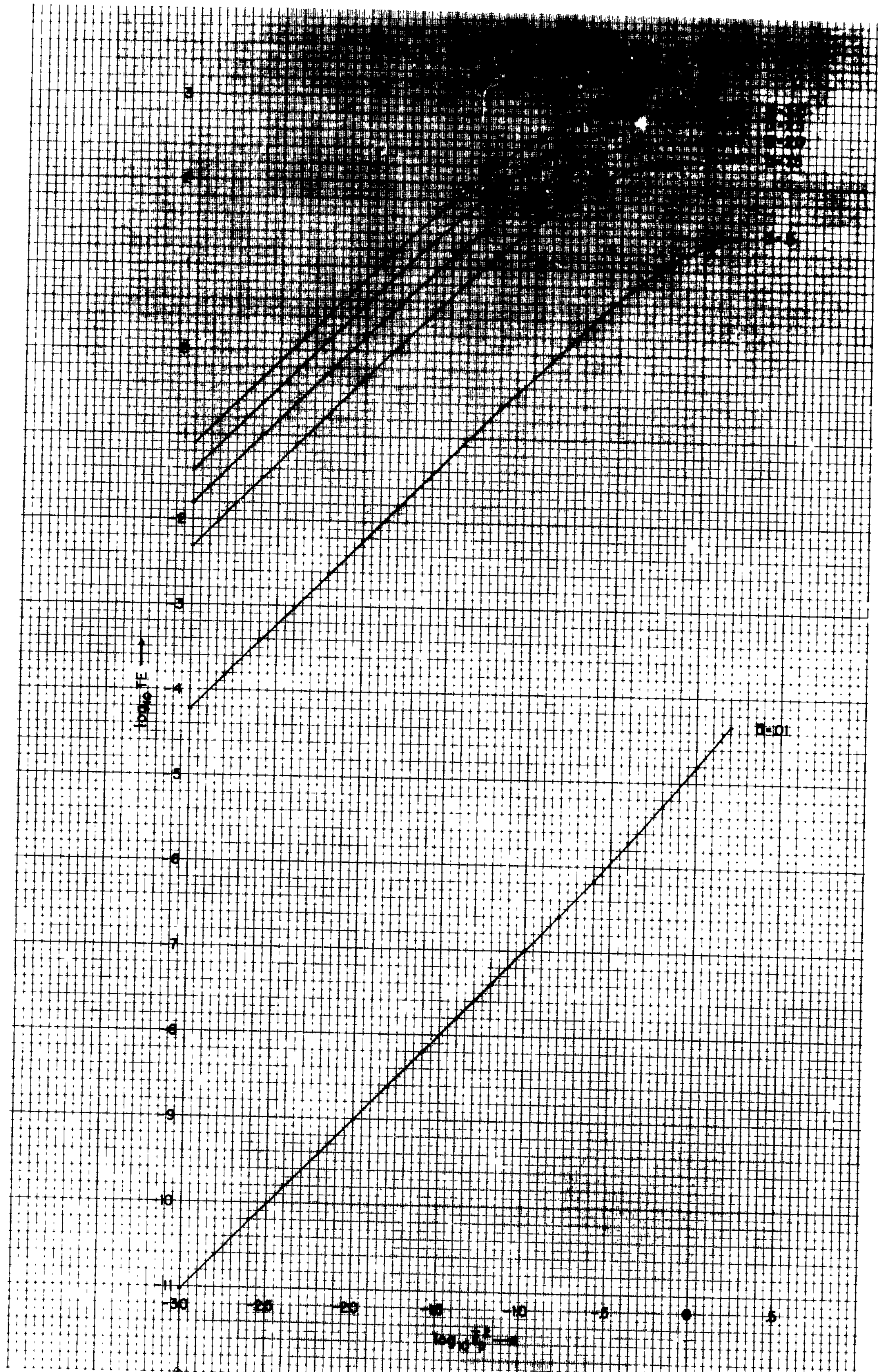

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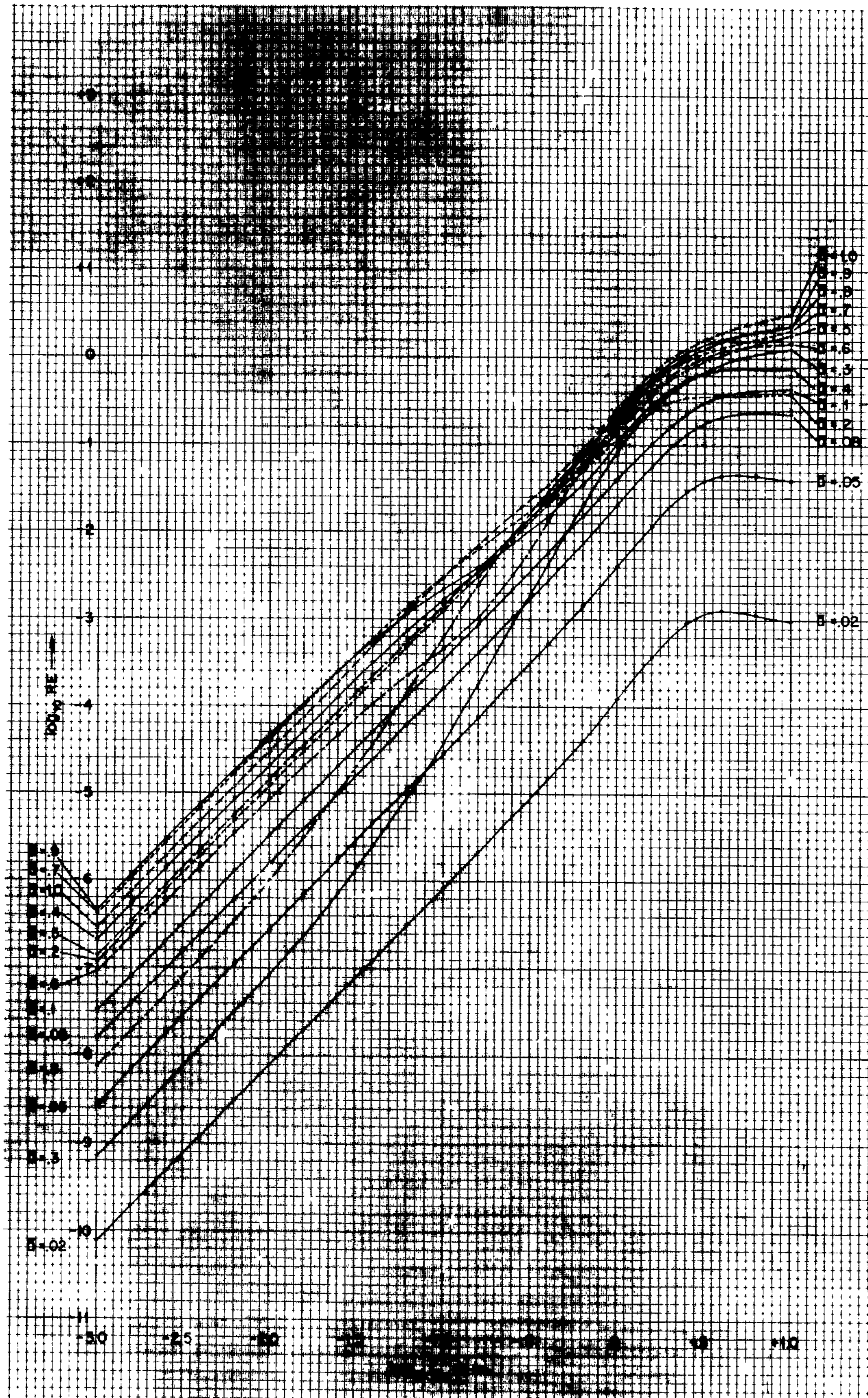

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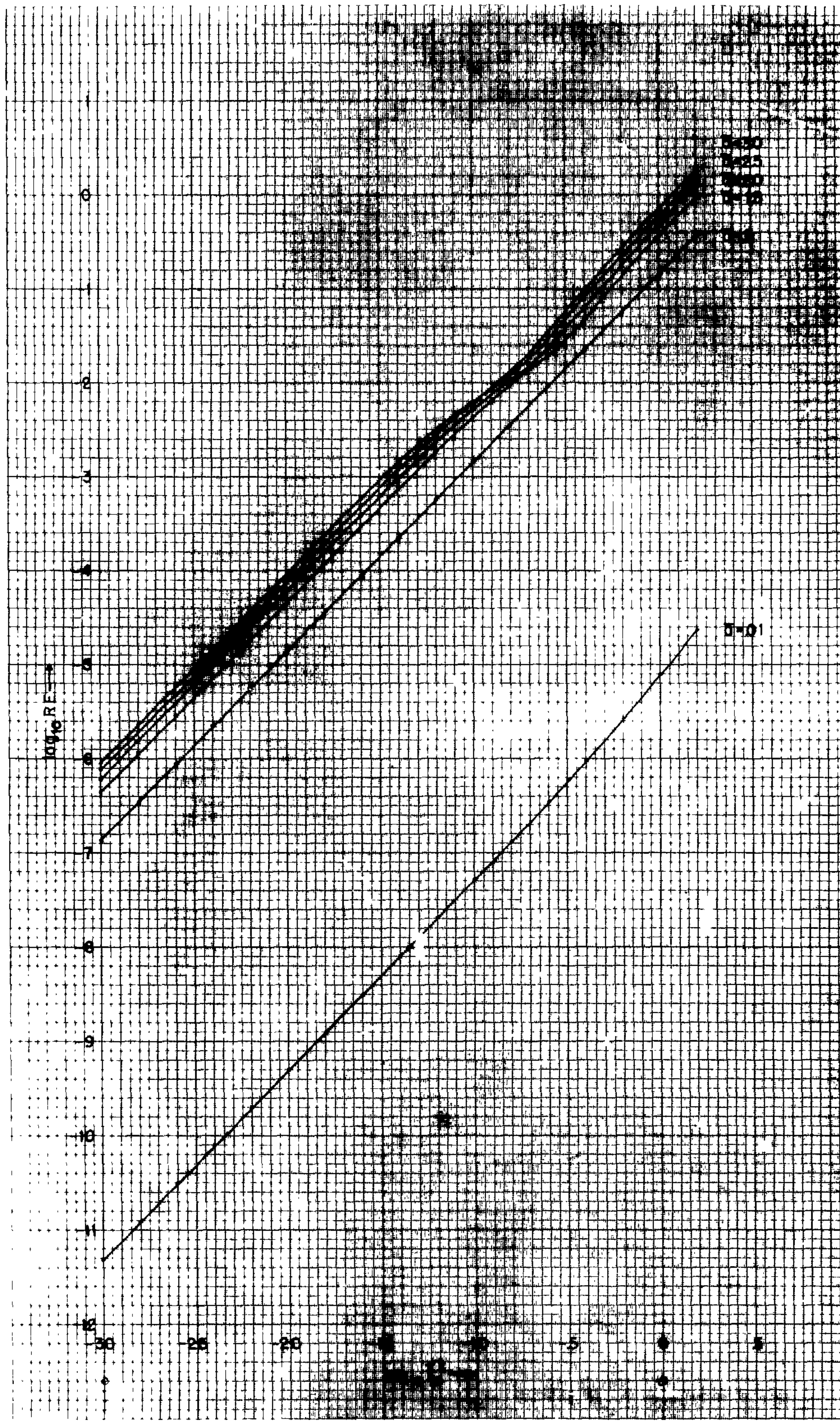


Figure 8

Figure 9

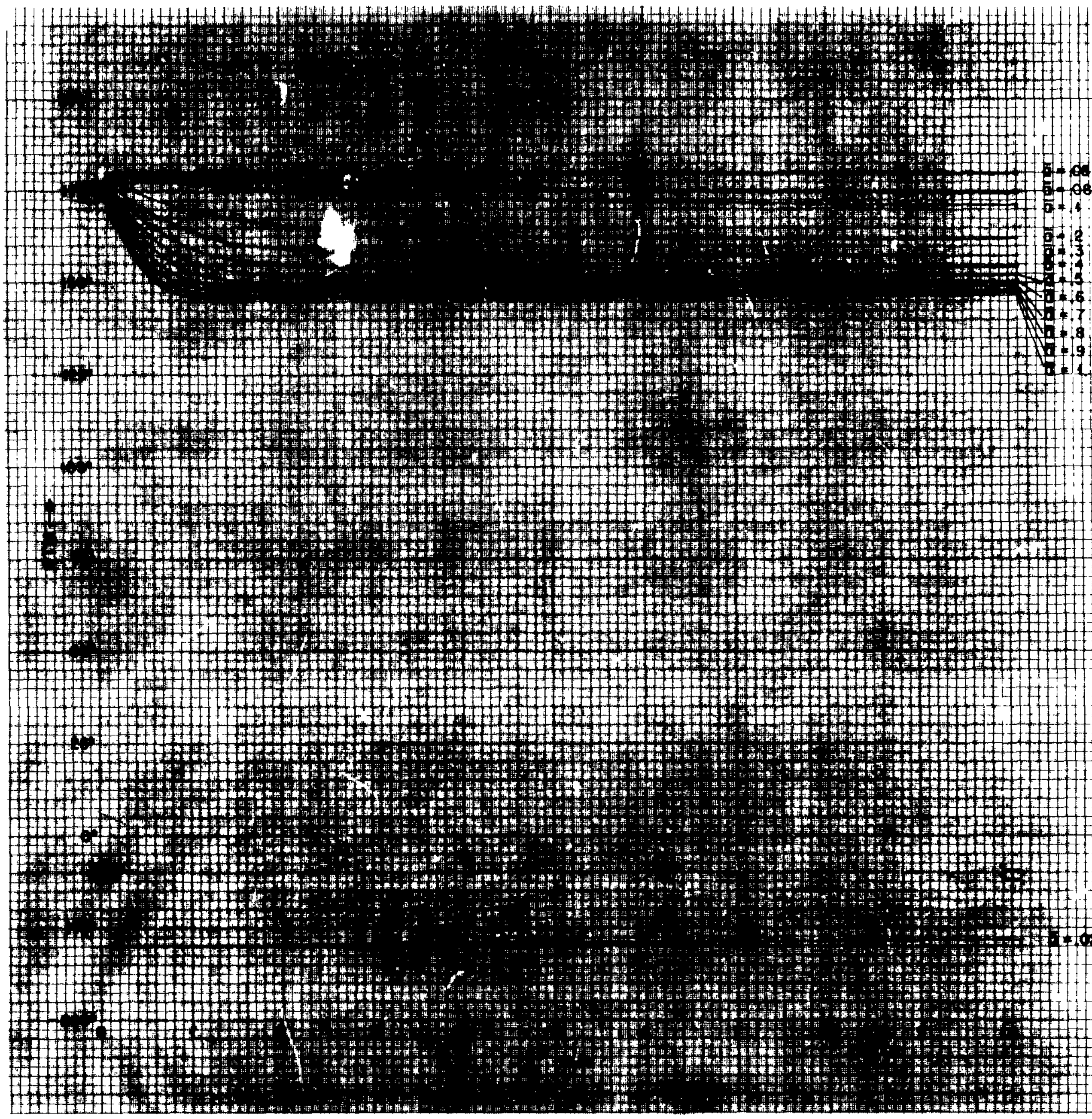
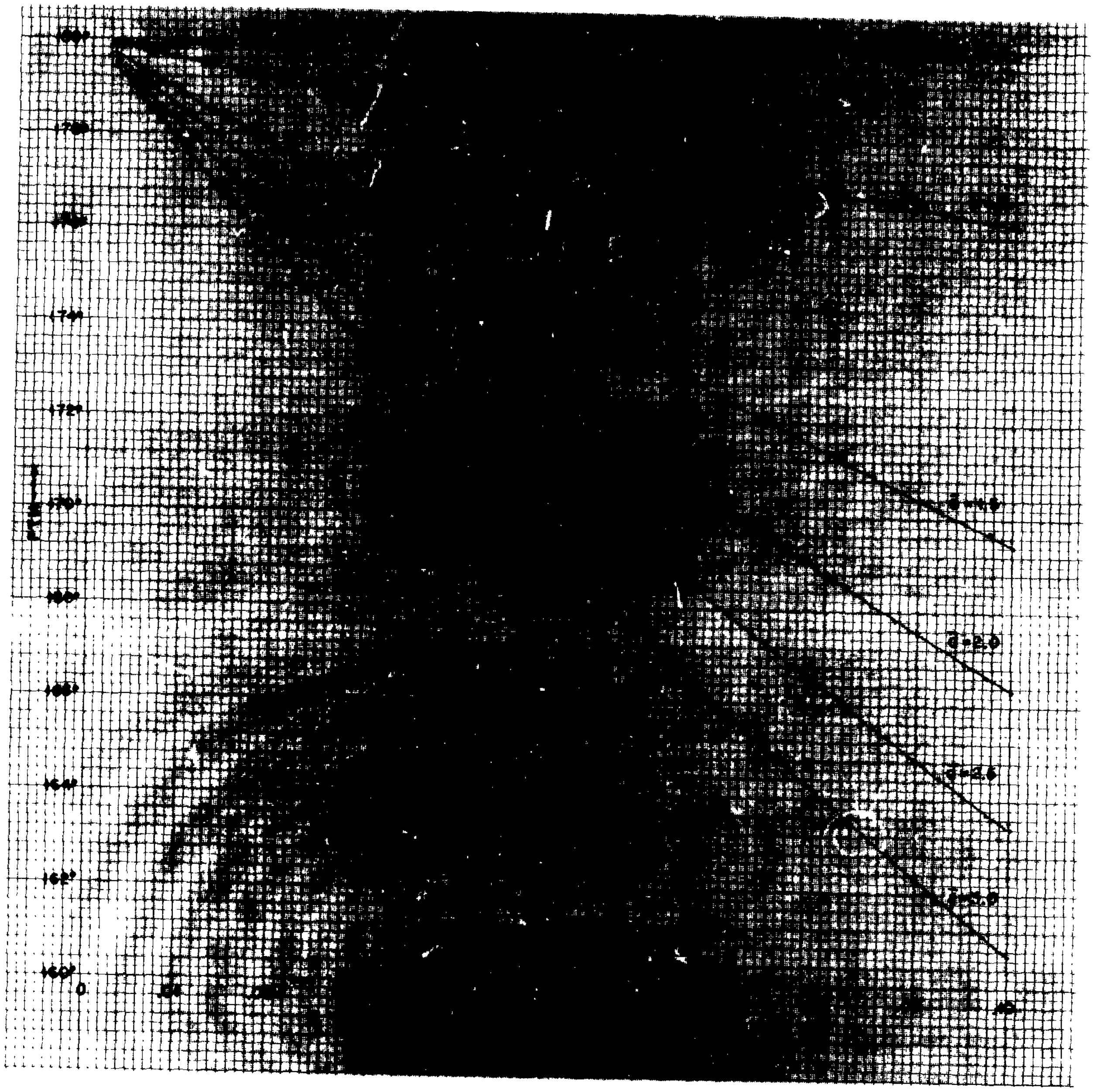


Figure 1C



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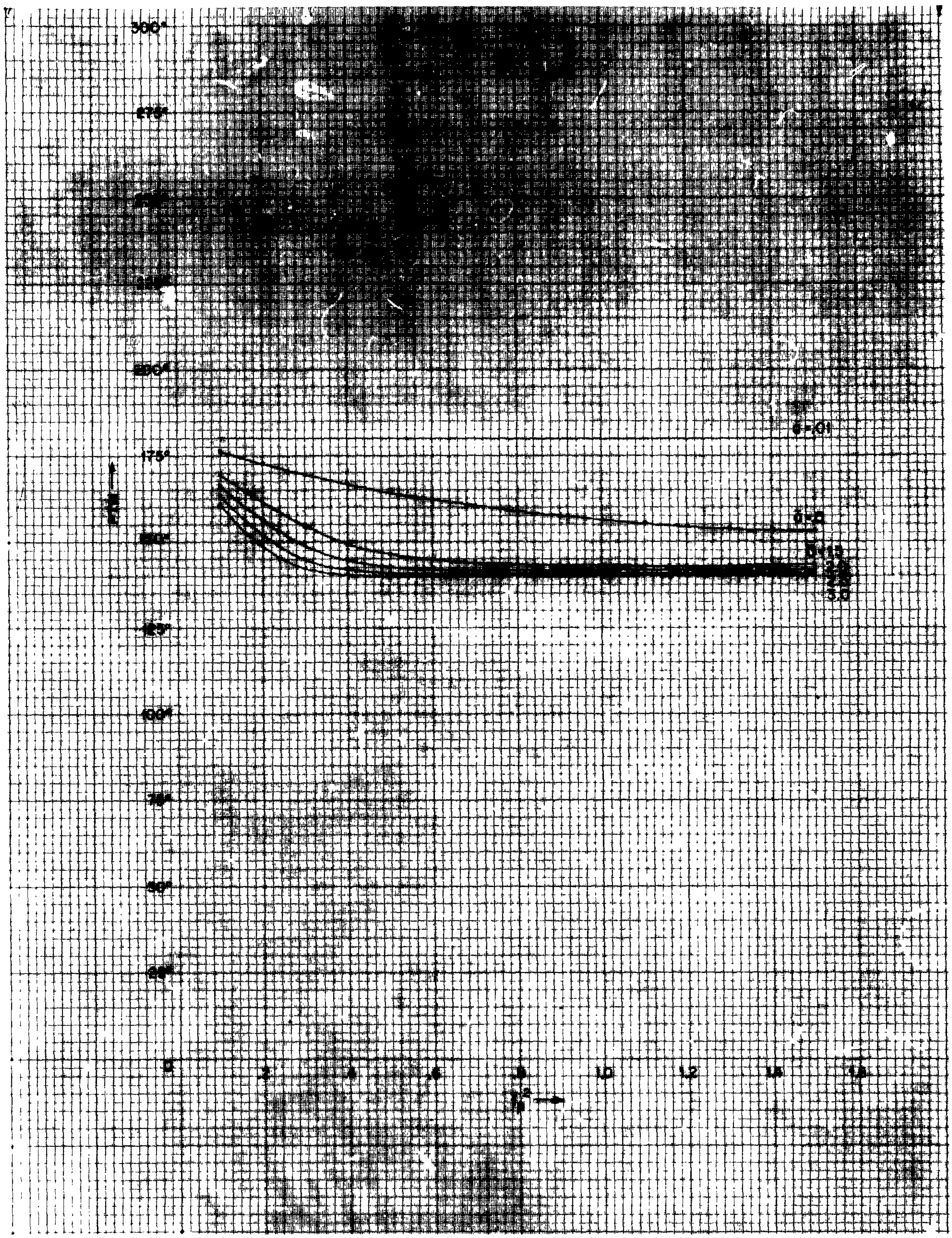

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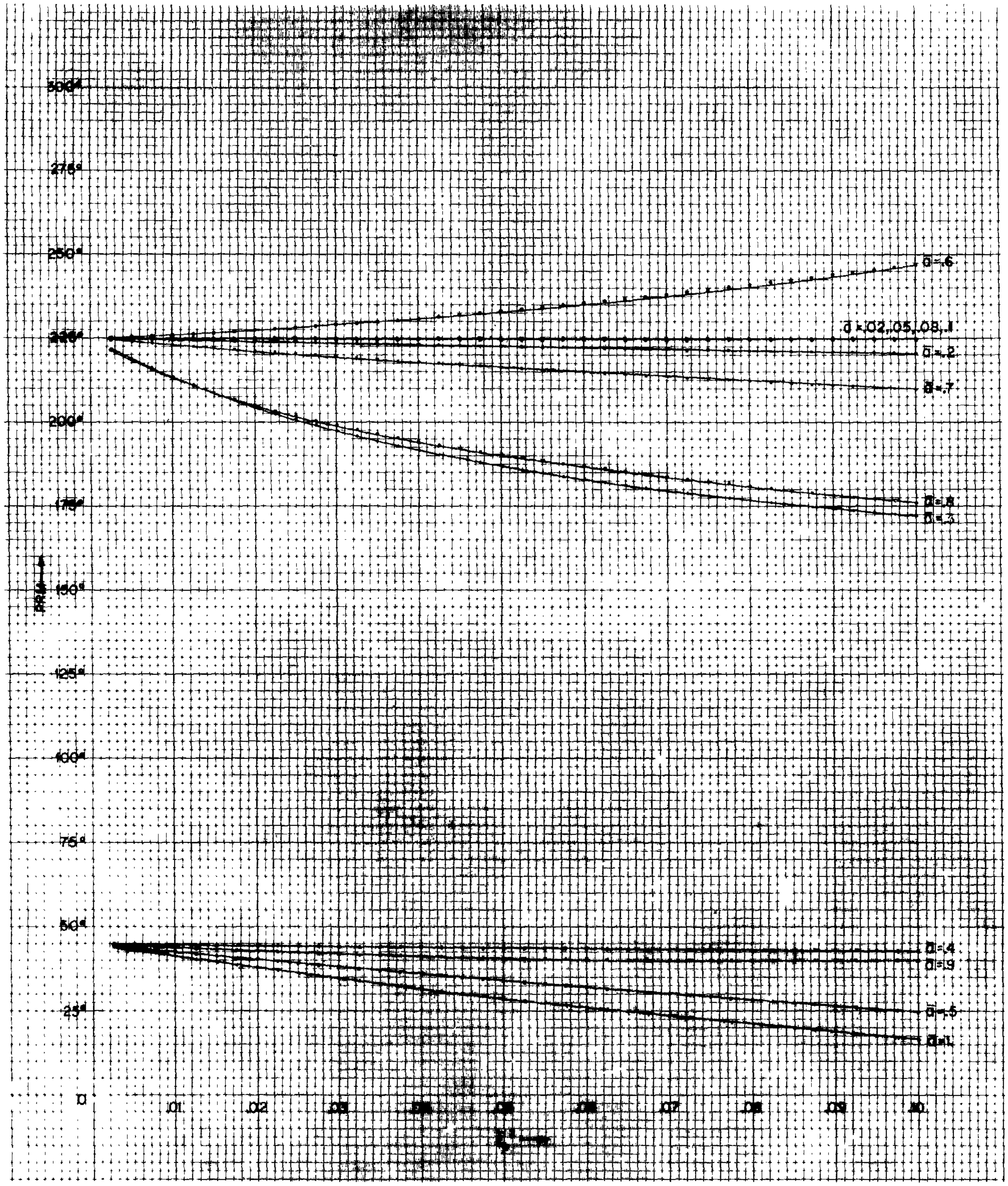
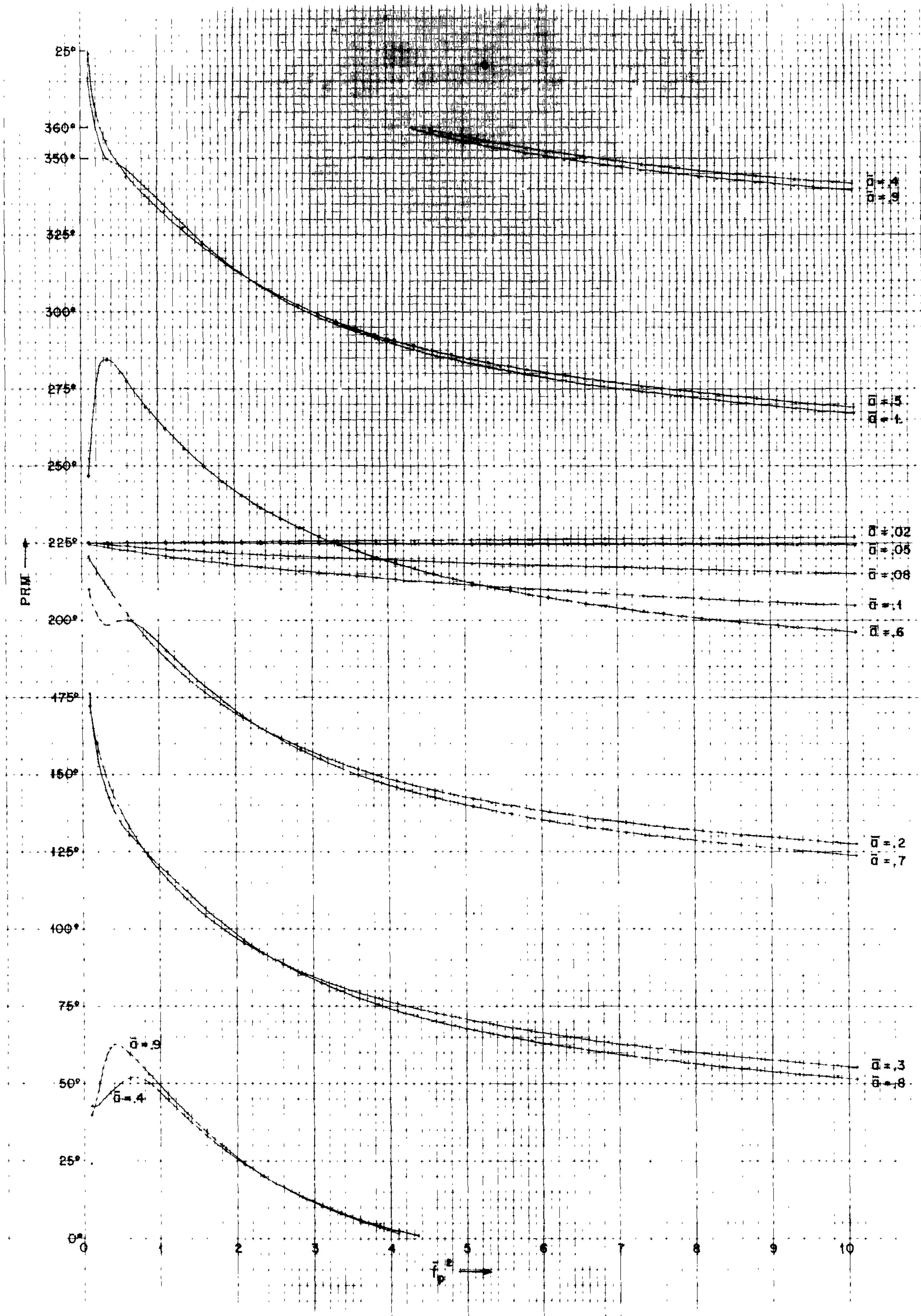


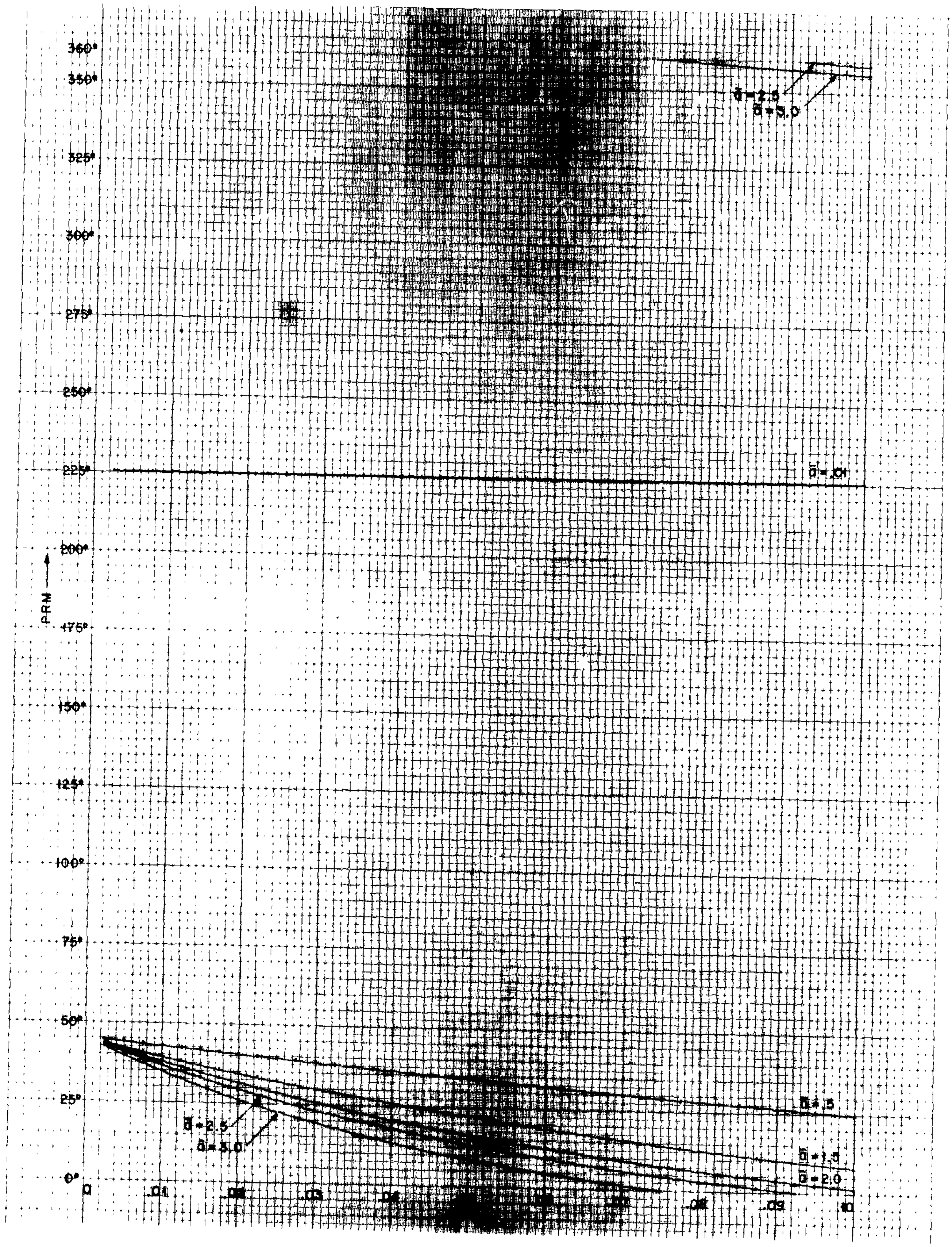
300°
275°
250°
225°
200°
175°
150°
125°
100°
75°
50°
25°
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
δ̄=.6
δ̄=.02,.05,.08,.1
δ̄=.2
δ̄=.7
δ̄=.8
δ̄=.3
δ̄=.4
δ̄=.9
δ̄=.5
δ̄=1.

Figure 13



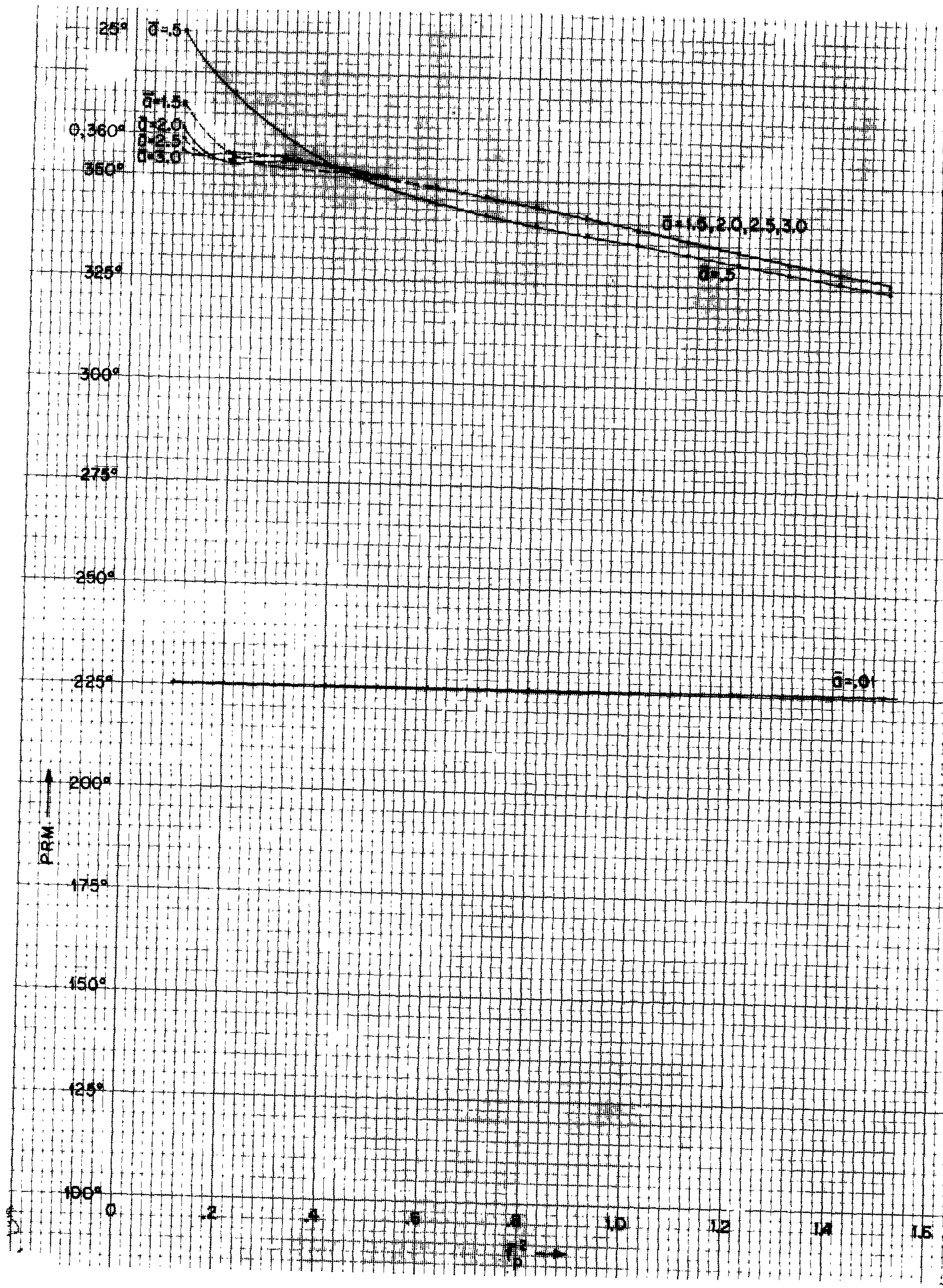
25°
360°
350°
325°
300°
275°
250°
225°
200°
175°
150°
125°
100°
75°
50°
25°
0°
PRM
$\bar{a}$ = .4
$\bar{a}$ = .9
$\bar{a}$ = .5
$\bar{a}$ = 1.
$\bar{a}$ = .02
$\bar{a}$ = .05
$\bar{a}$ = .08
$\bar{a}$ = .1
$\bar{a}$ = .6
$\bar{a}$ = .2
$\bar{a}$ = .7
$\bar{a}$ = .3
$\bar{a}$ = .8
$\bar{a}$ = .9
$\bar{a}$ = .4
0
1
2
3
4
5
6
7
8
9
10
f_p^2

Figure 14



360°
350°
325°
300°
275°
250°
225°
200°
PRM
175°
150°
125°
100°
75°
50°
25°
0°
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
$\bar{a}$ = 2.5
$\bar{a}$ = 3.0
$\bar{a}$ = .01
$\bar{a}$ = .5
$\bar{a}$ = 1.5
$\bar{a}$ = 2.0
$\bar{a}$ = 2.5
$\bar{a}$ = 3.0

Figure 15



25°
0,360°
350°
325°
300°
275°
250°
225°
200°
175°
150°
125°
100°
PRM
ā=.5
ā=1.5
ā=2.0
ā=2.5
ā=3.0
ā=1.5,2.0,2.5,3.0
ā=.5
ā=.0
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6

Figure 15

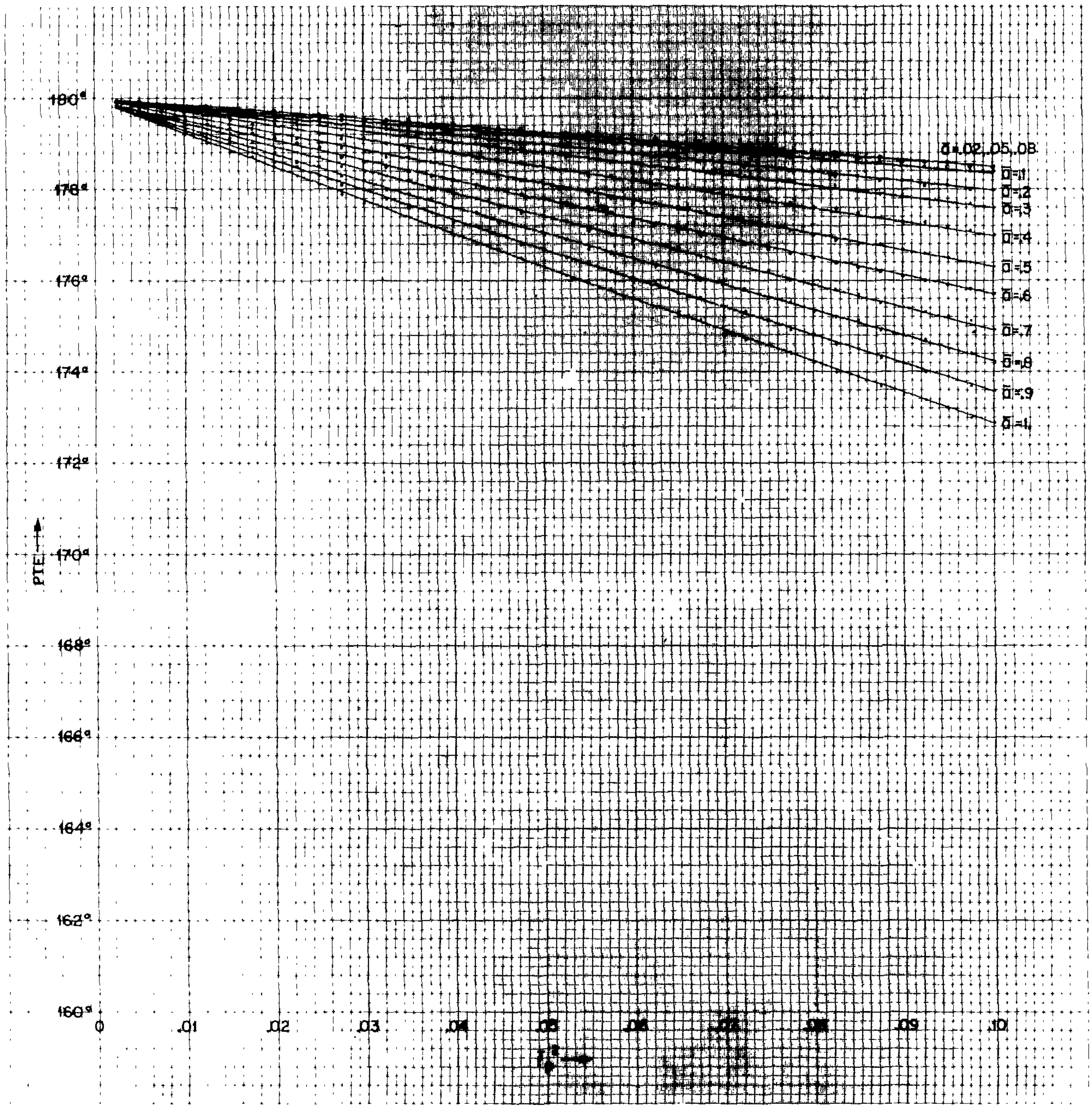
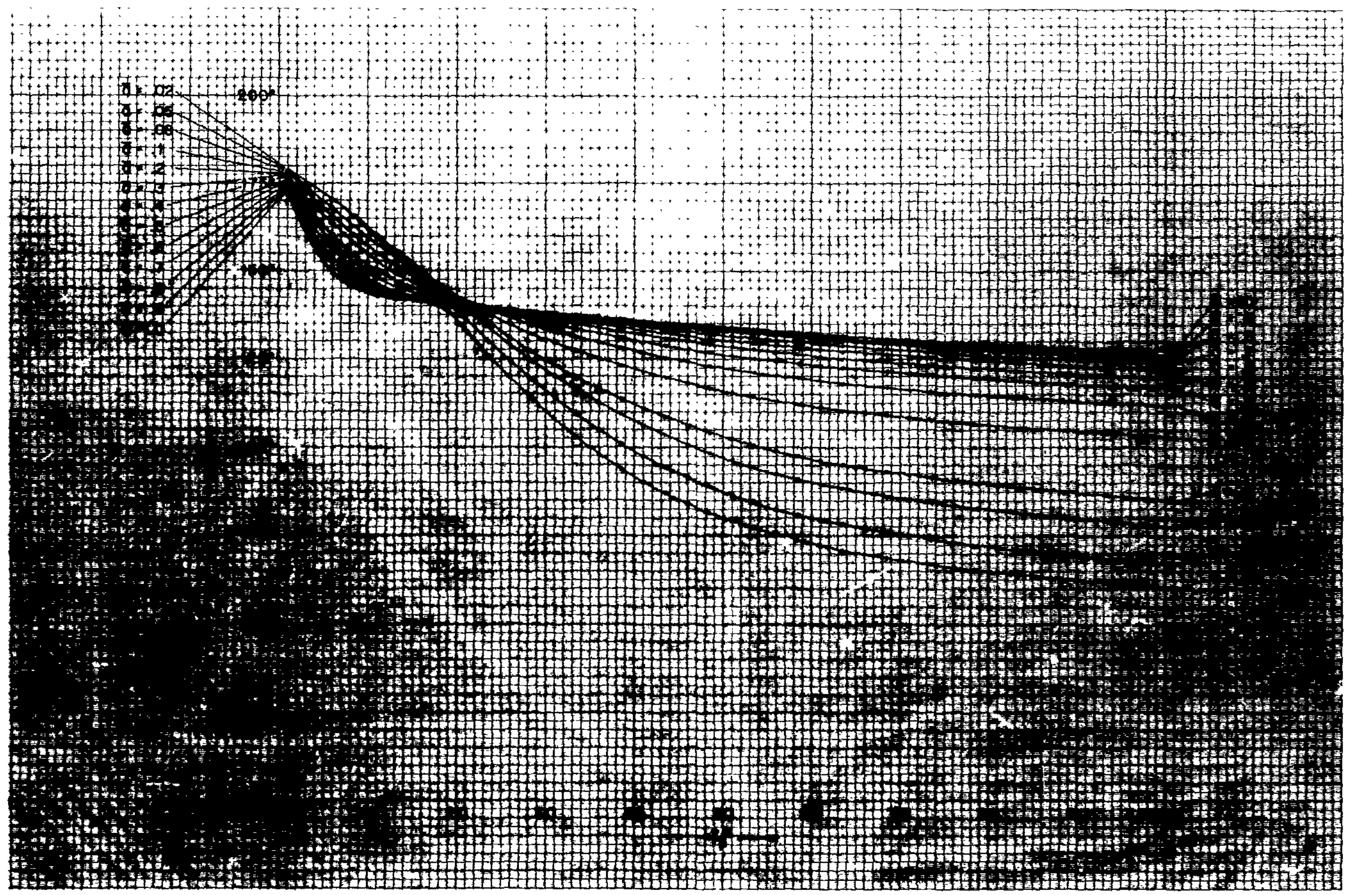

PTE
180°
178°
176°
174°
172°
170°
168°
166°
164°
162°
160°
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
δ=.02,.05,.08
δ=.1
δ=.2
δ=.3
δ=.4
δ=.5
δ=.6
δ=.7
δ=.8
δ=.9
δ=1.

Figure 17



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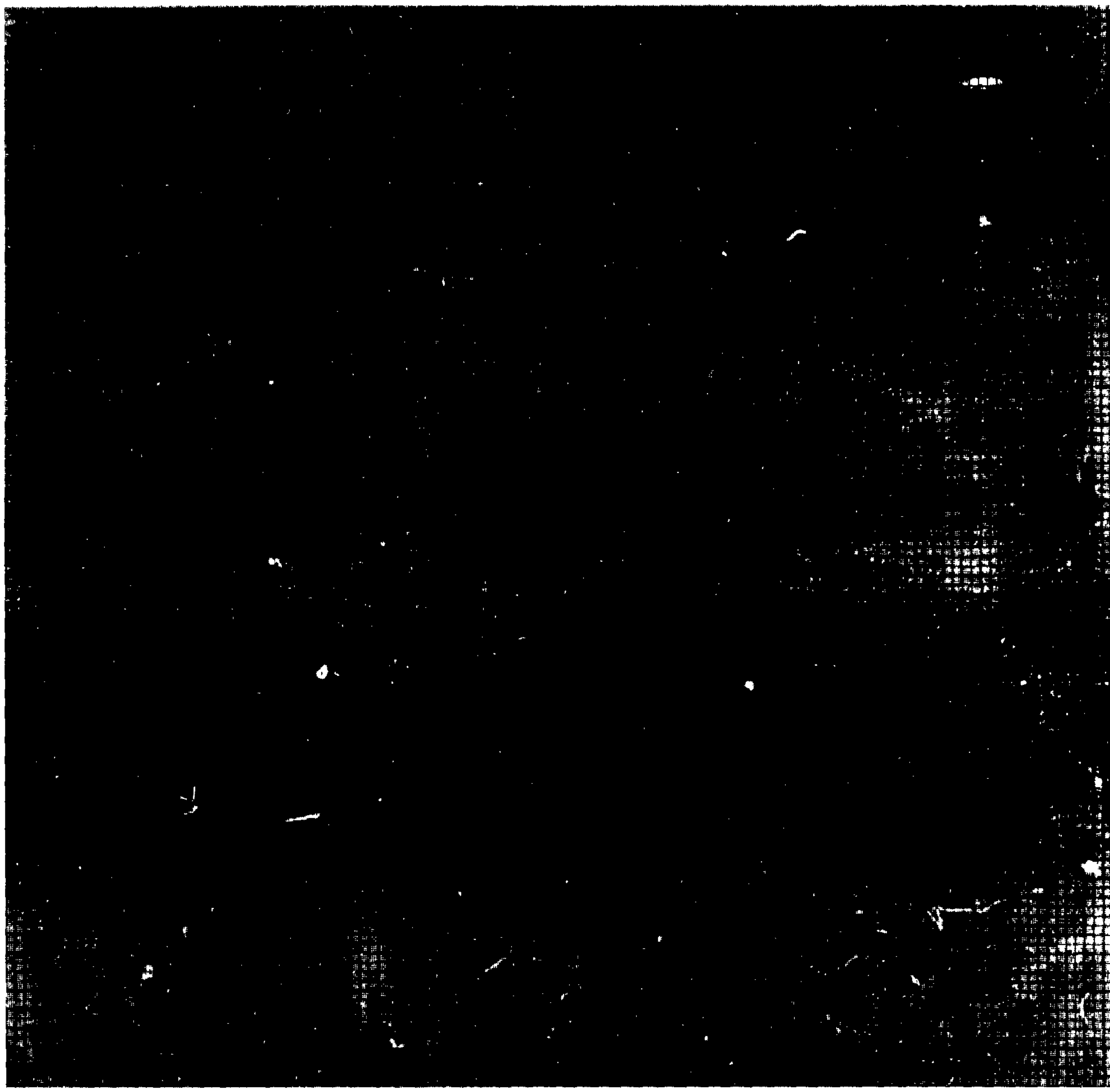


Figure 19

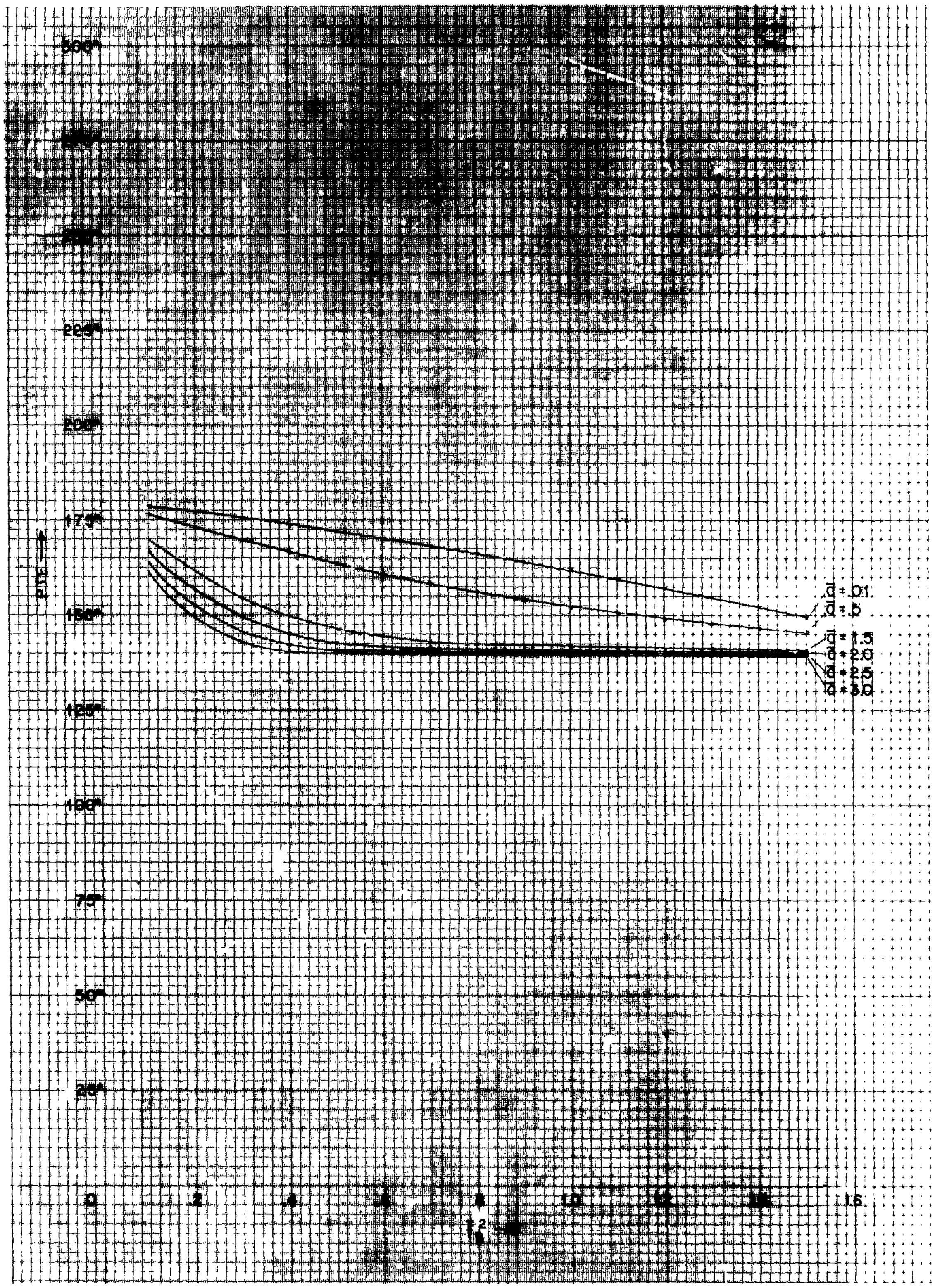


Figure 20

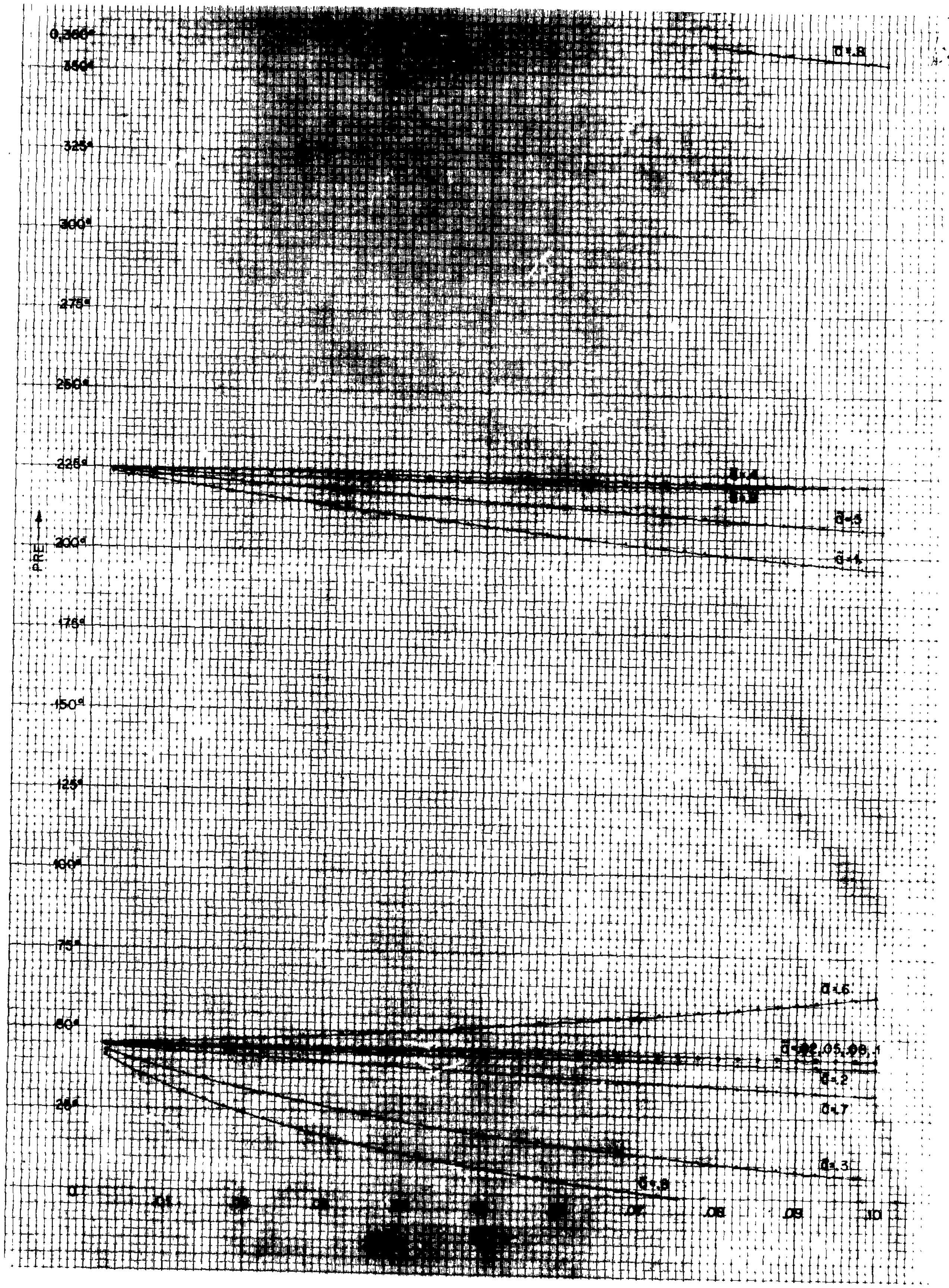
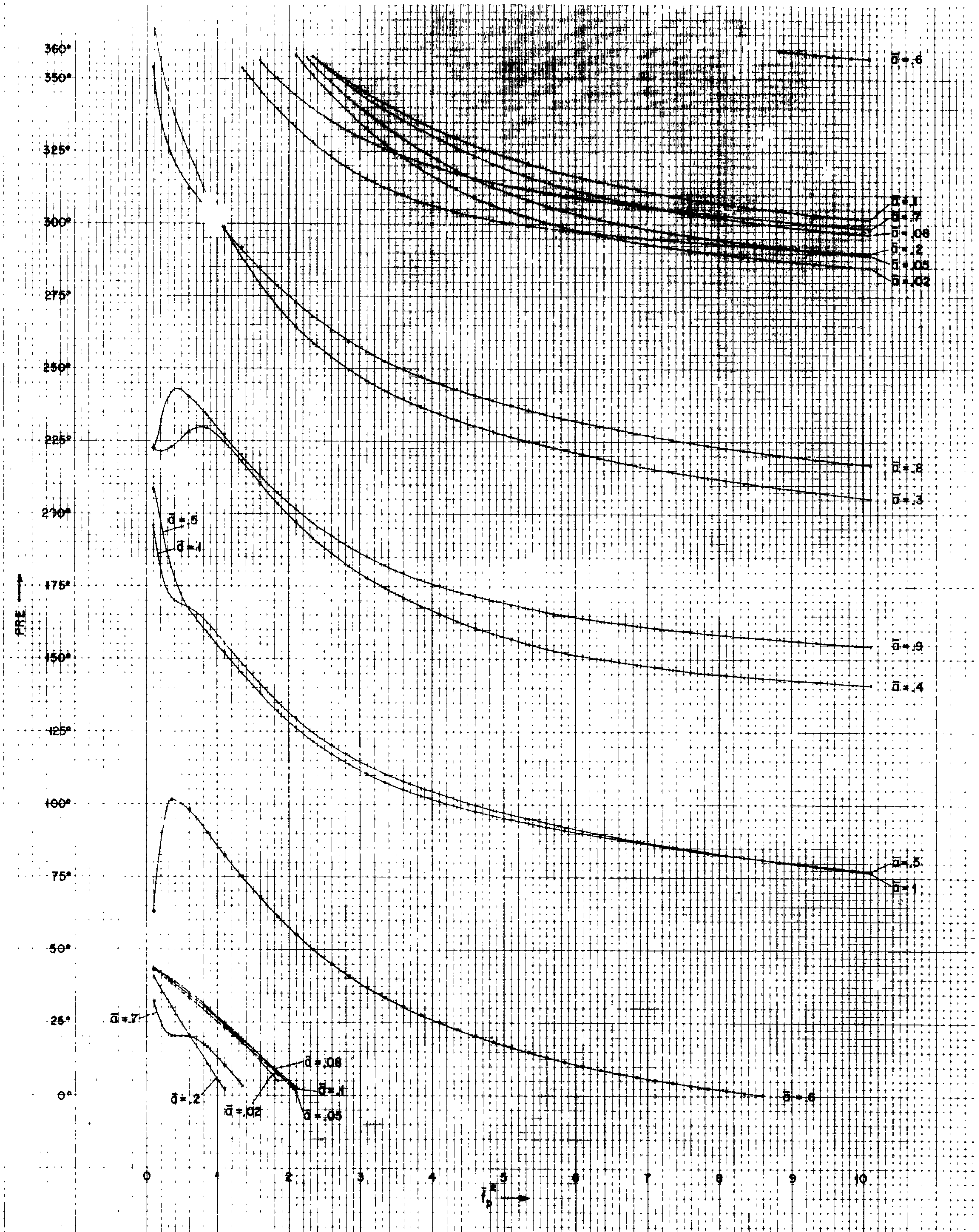


Figure 21



360°
350°
325°
300°
275°
250°
225°
200°
175°
150°
125°
100°
75°
50°
25°
0°
PRE
0
1
2
3
4
5
6
7
8
9
10
$\bar{f}_p^2$
$\bar{a}$ = .6
$\bar{a}$ = .1
$\bar{a}$ = .7
$\bar{a}$ = .08
$\bar{a}$ = .2
$\bar{a}$ = .05
$\bar{a}$ = .02
$\bar{a}$ = .8
$\bar{a}$ = .3
$\bar{a}$ = .5
$\bar{a}$ = .1
$\bar{a}$ = .9
$\bar{a}$ = .4
$\bar{a}$ = .5
$\bar{a}$ = 1
$\bar{a}$ = .7
$\bar{a}$ = .08
$\bar{a}$ = .1
$\bar{a}$ = .2
$\bar{a}$ = .02
$\bar{a}$ = .05
$\bar{a}$ = .6

Figure 22

Figure 23

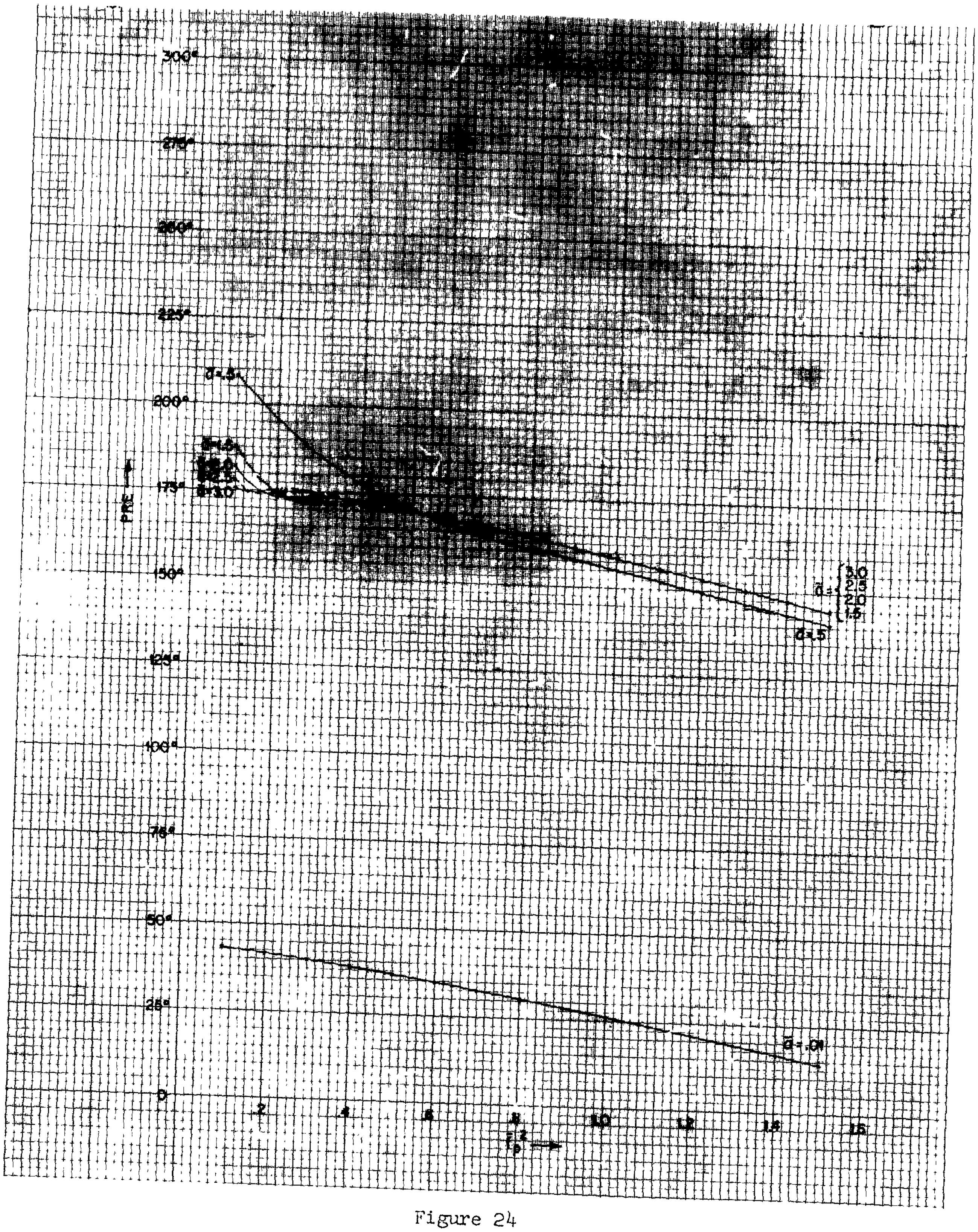


Figure 24

$\bar{\nu} = .1448$

Figures 25 - 78

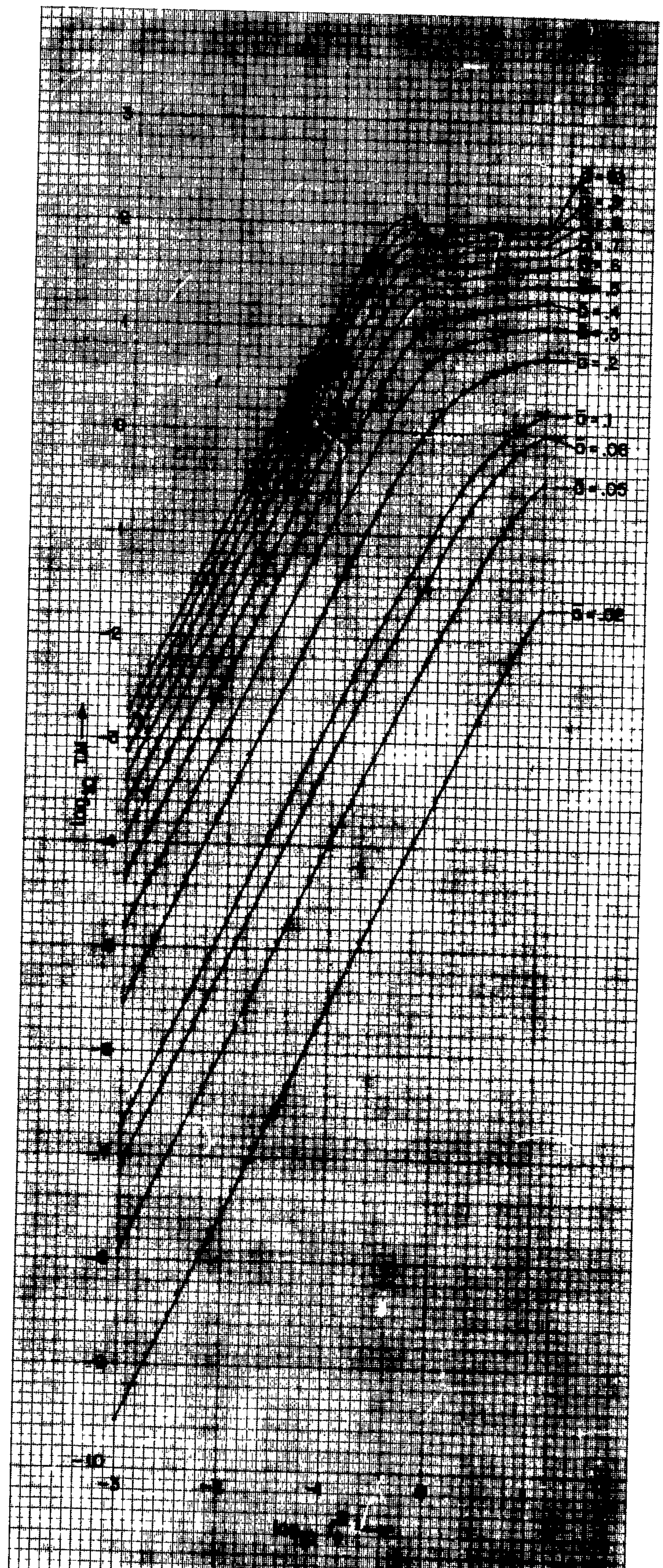


Figure 2)

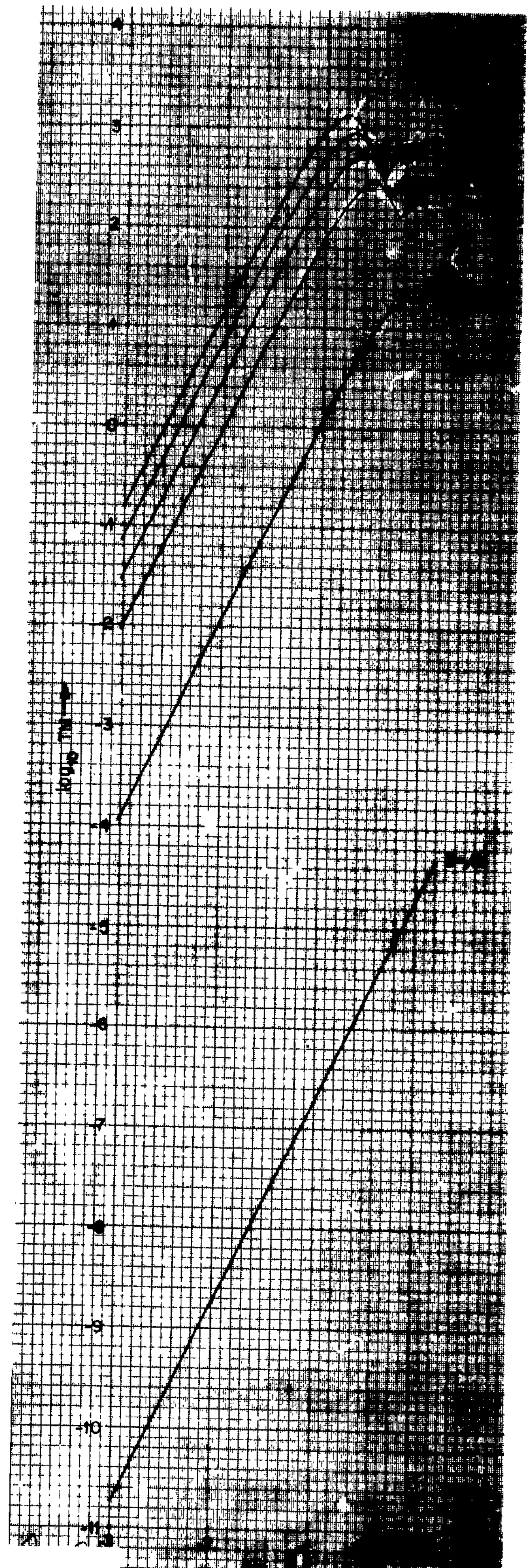
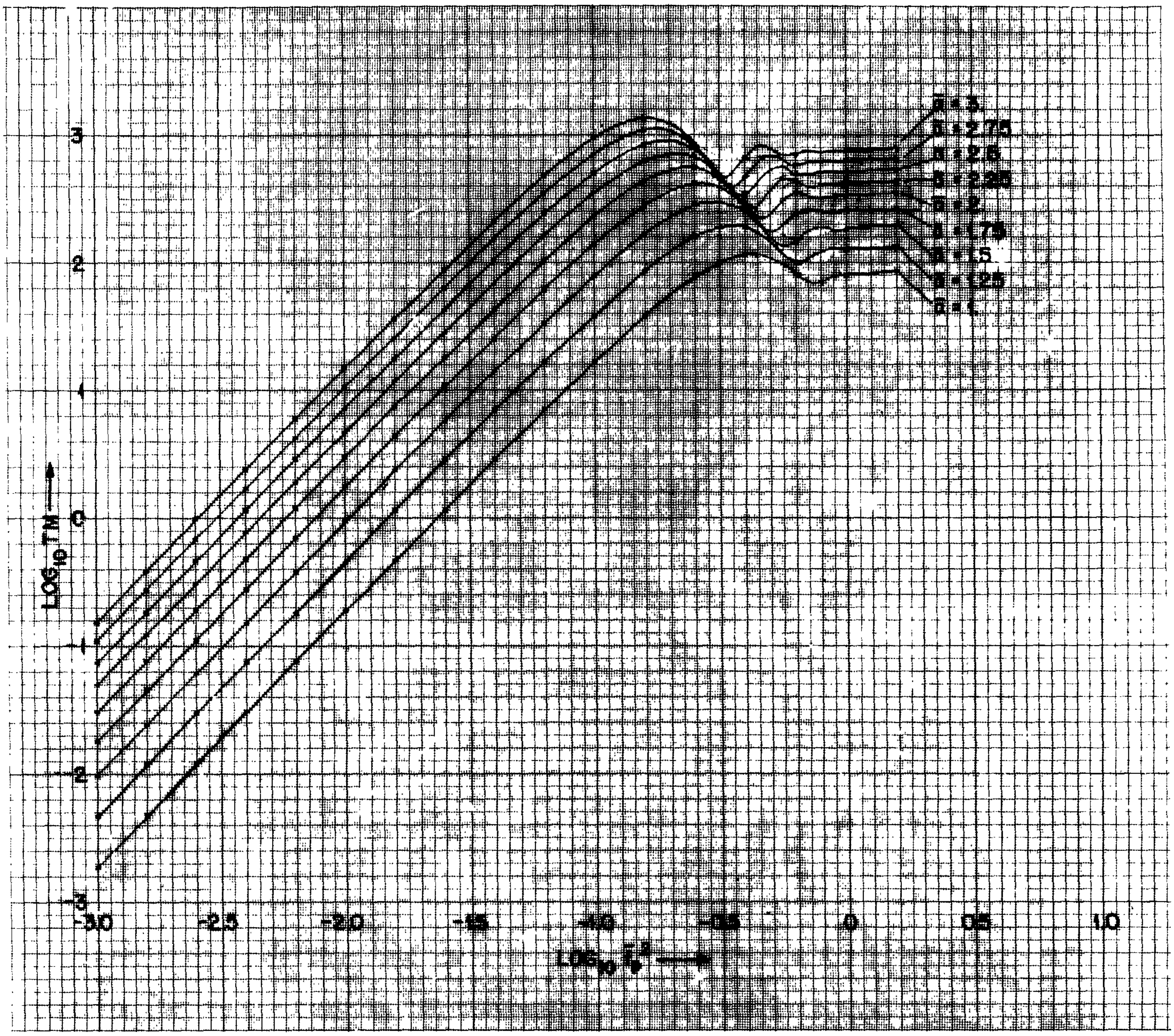
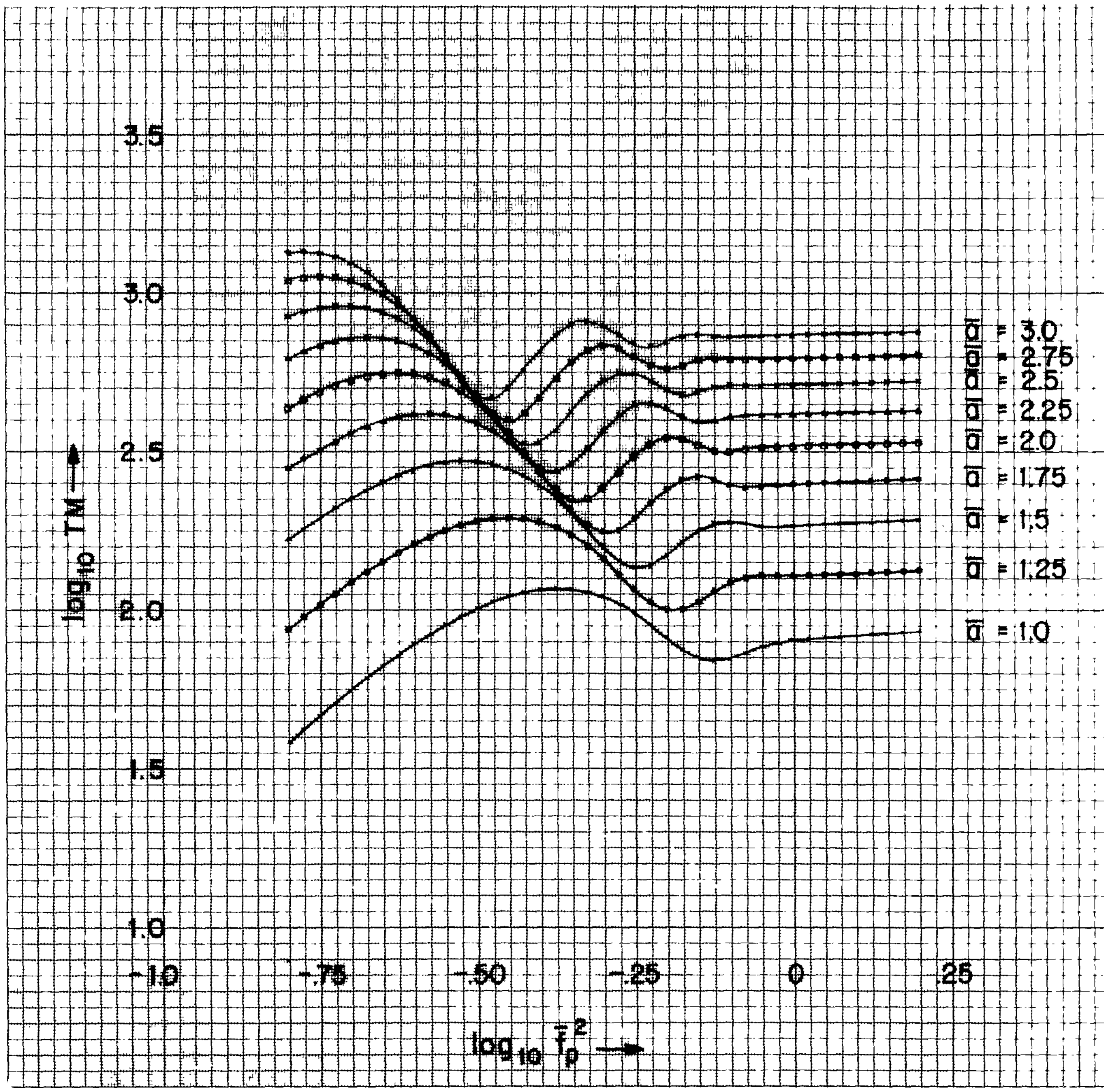


Figure 26



S = 3
S = 2.75
S = 2.5
S = 2.25
S = 2
S = 1.75
S = 1.5
S = 1.25
S = 1
3
2
1
0
-1
-2
-3
-3.0
-2.5
-2.0
-1.5
-1.0
-0.5
0
0.5
1.0

Figure 27



3.5
3.0
2.5
2.0
1.5
1.0
$\log_{10}$ TM
-1.0
-.75
-.50
-.25
0
.25
$\log_{10} \bar{f}_p^2$
$\bar{a}$ = 3.0
$\bar{a}$ = 2.75
$\bar{a}$ = 2.5
$\bar{a}$ = 2.25
$\bar{a}$ = 2.0
$\bar{a}$ = 1.75
$\bar{a}$ = 1.5
$\bar{a}$ = 1.25
$\bar{a}$ = 1.0

Figure 28

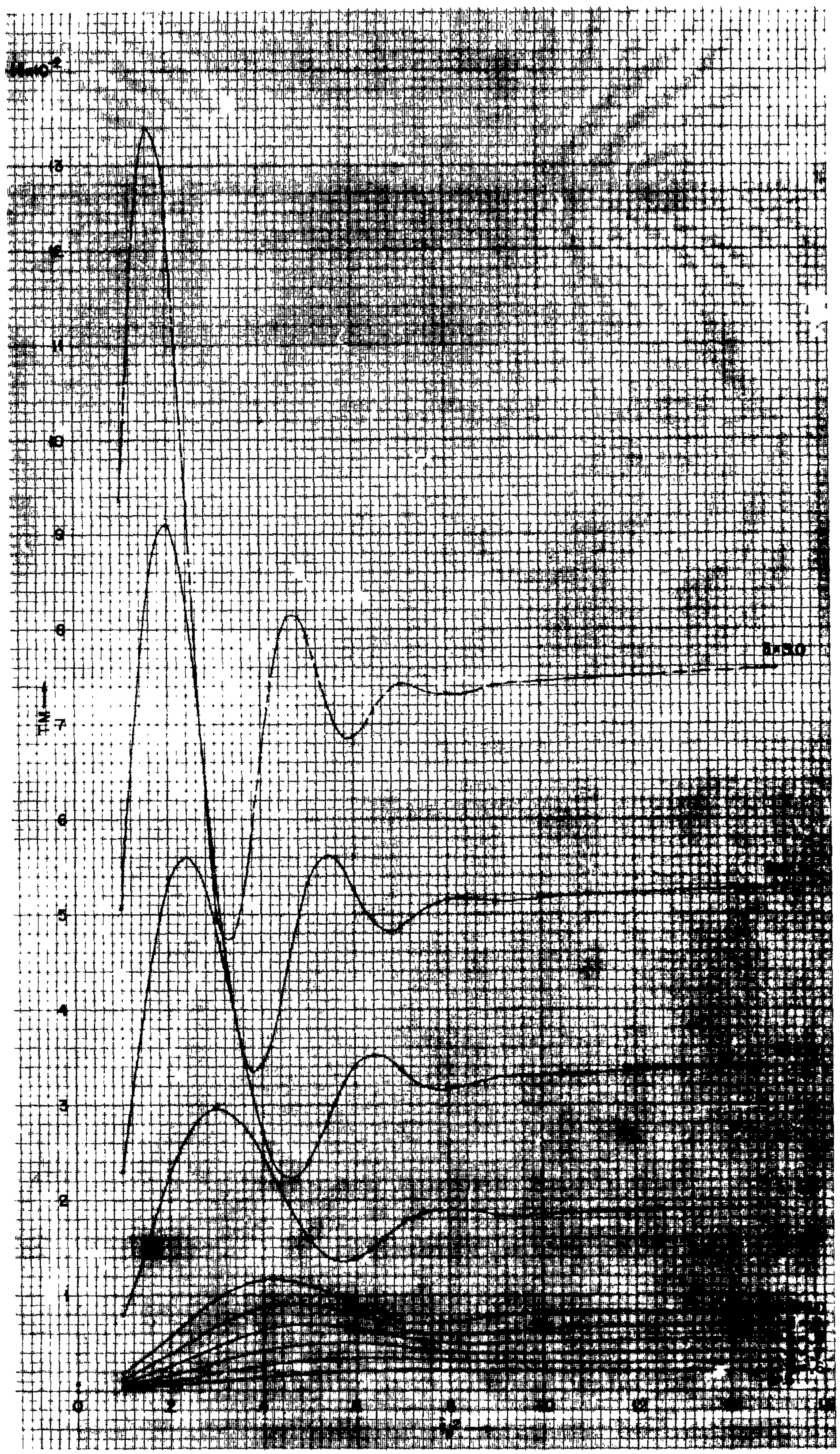


Figure 2y

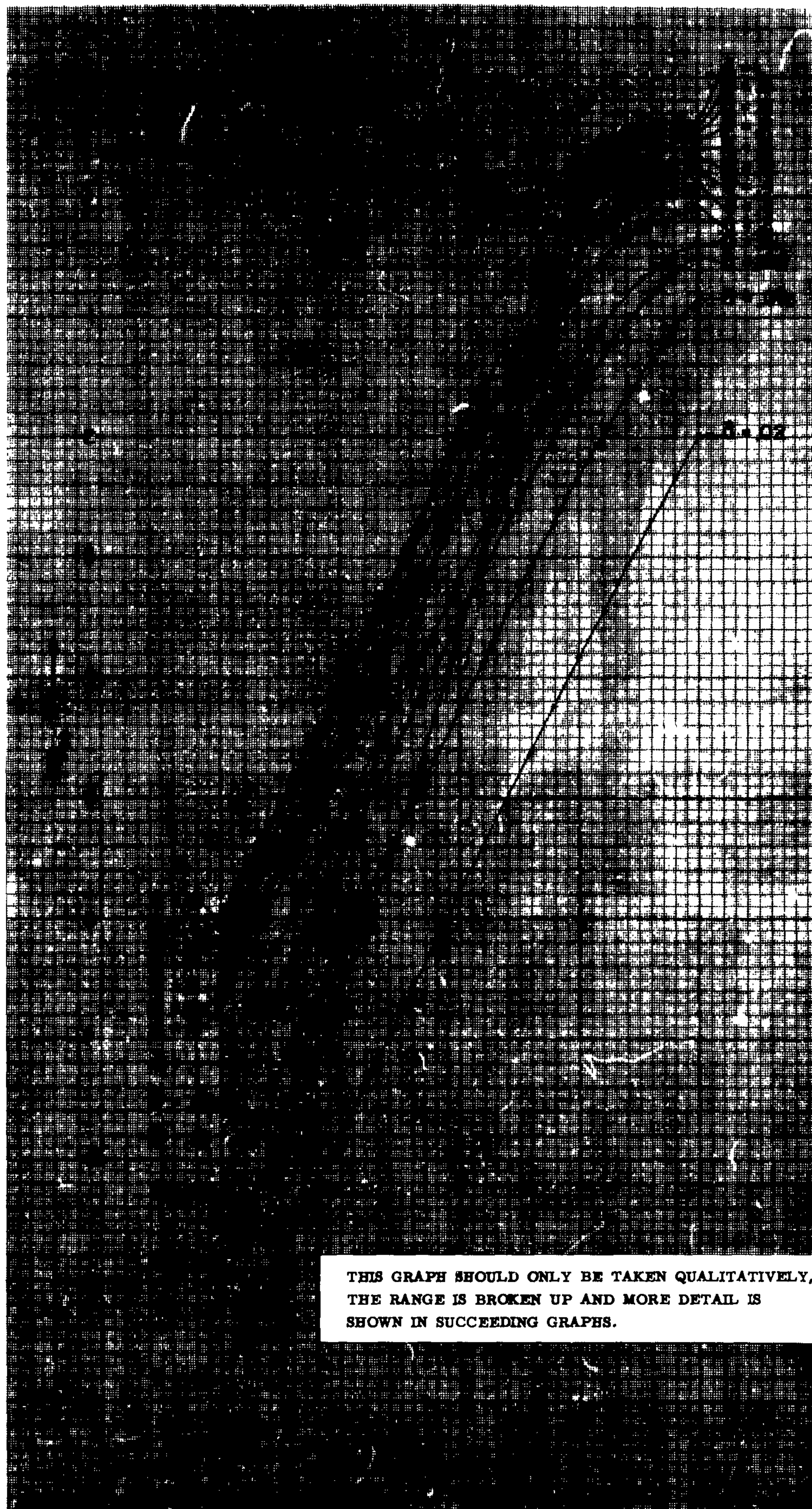
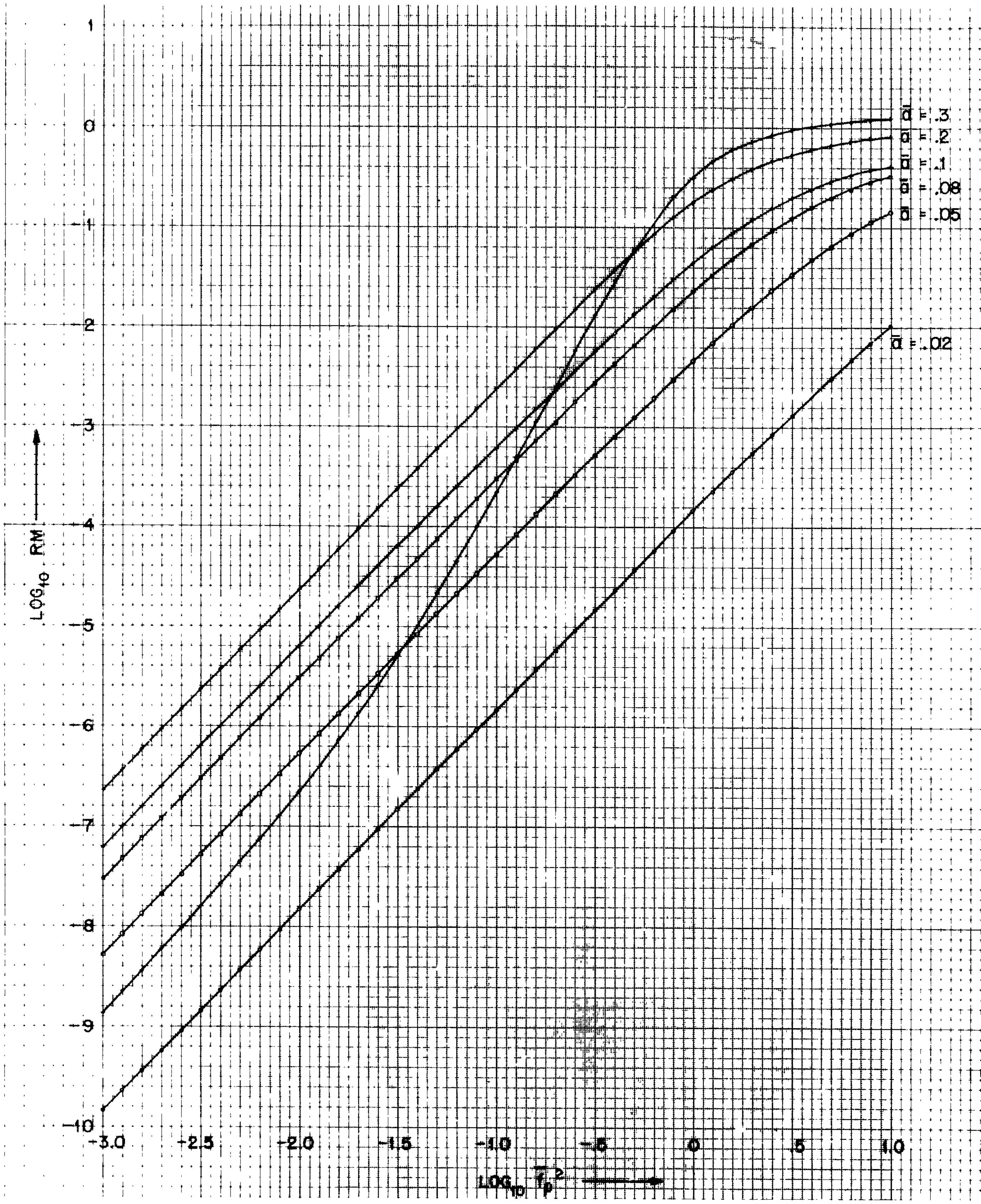

THIS GRAPH SHOULD ONLY BE TAKEN QUALITATIVELY,
THE RANGE IS BROKEN UP AND MORE DETAIL IS
SHOWN IN SUCCEEDING GRAPHS.

Figure 30



$\bar{a}$ = .3
$\bar{a}$ = .2
$\bar{a}$ = .1
$\bar{a}$ = .08
$\bar{a}$ = .05
$\bar{a}$ = .02
LOG_{10} RM
LOG_{10} $\bar{f}_p^2$
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-3.0
-2.5
-2.0
-1.5
-1.0
-.5
0
.5
1.0

Figure 31

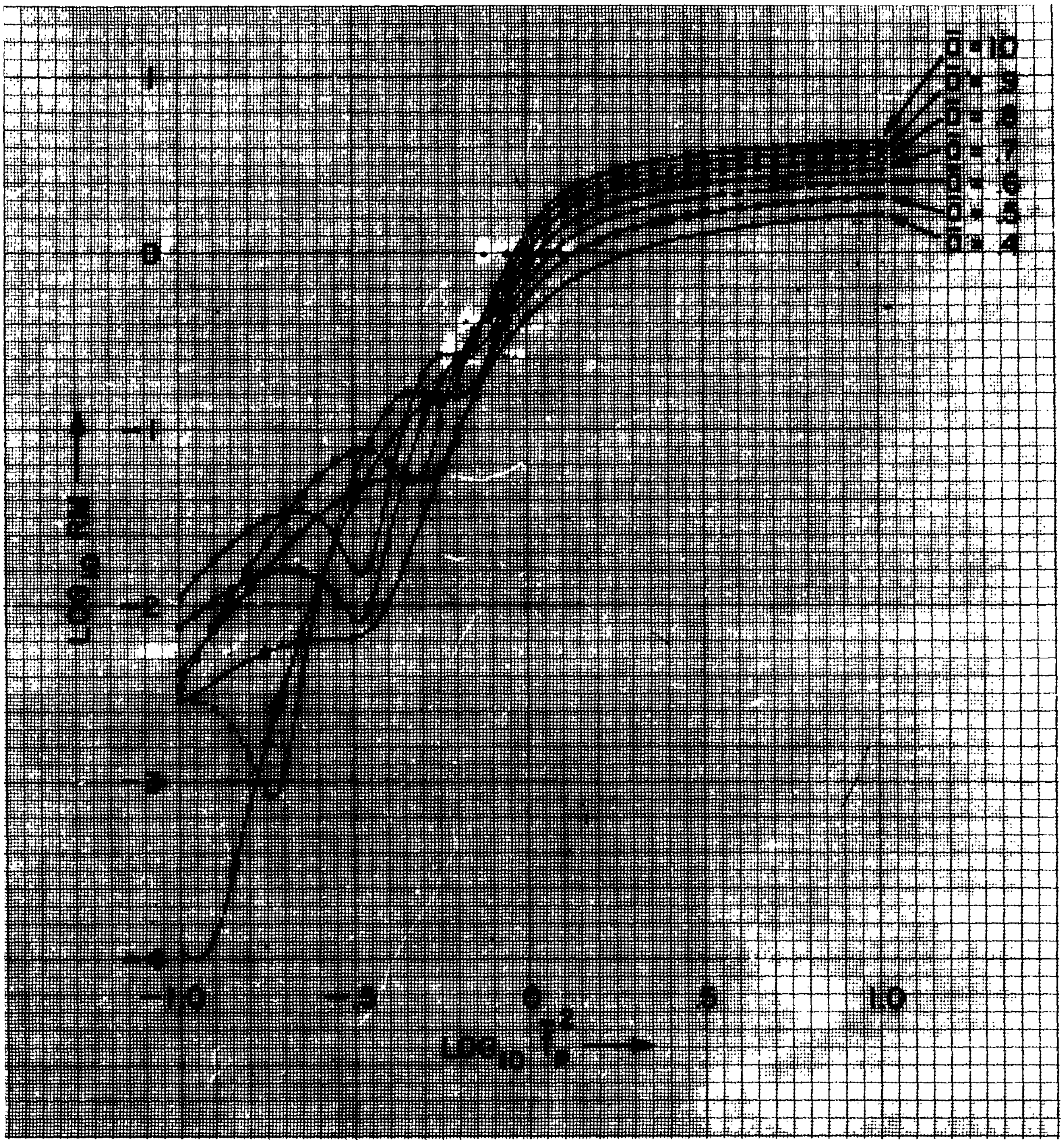
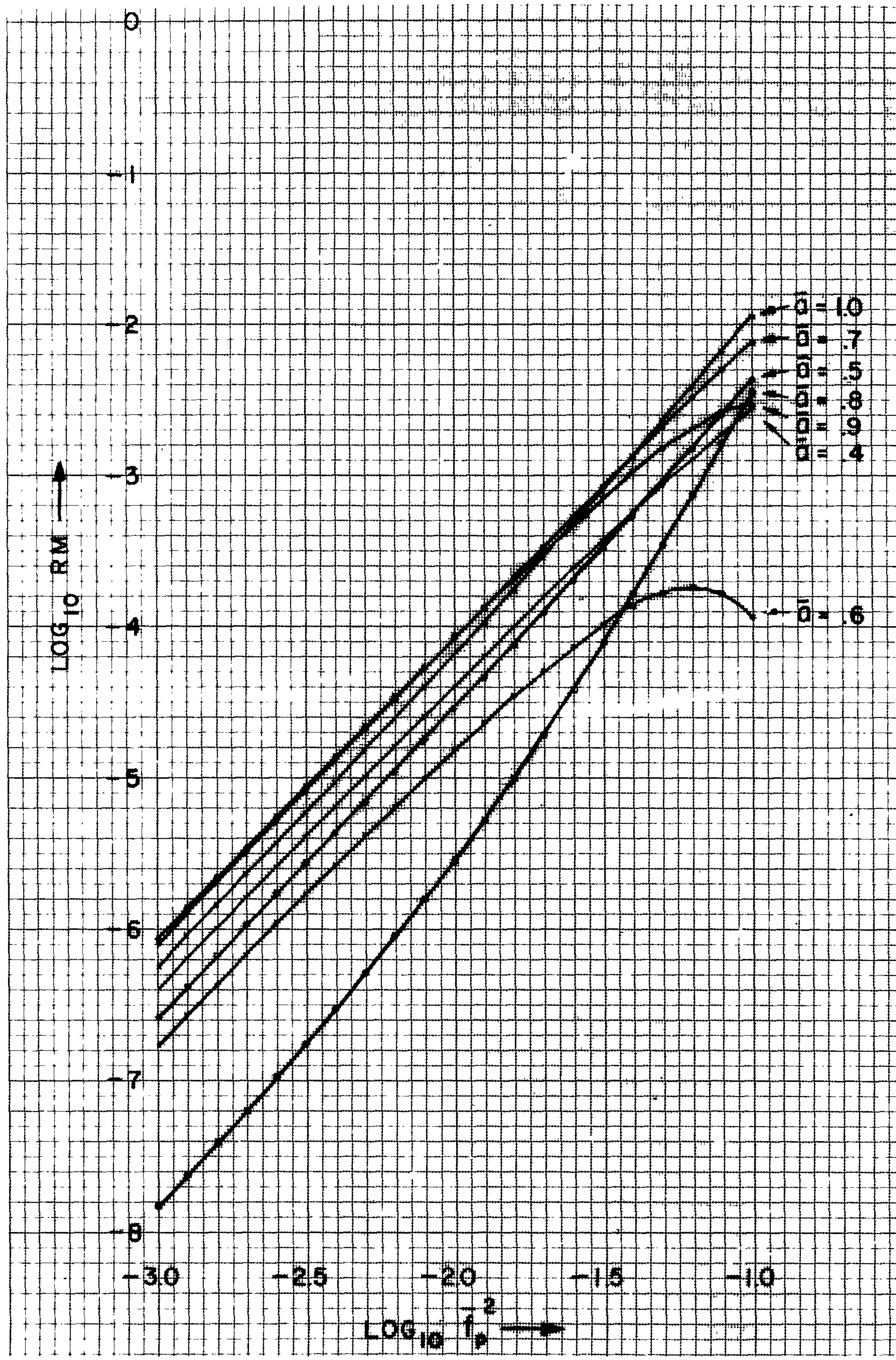


Figure 32



0
-1
-2
-3
-4
-5
-6
-7
-8
-3.0
-2.5
-2.0
-1.5
-1.0
LOG₁₀ RM
LOG₁₀ $\bar{f}_p^2$
$\bar{D}$ = 1.0
$\bar{D}$ = .7
$\bar{D}$ = .5
$\bar{D}$ = .8
$\bar{D}$ = .9
$\bar{D}$ = .4
$\bar{D}$ = .6

Figure 33

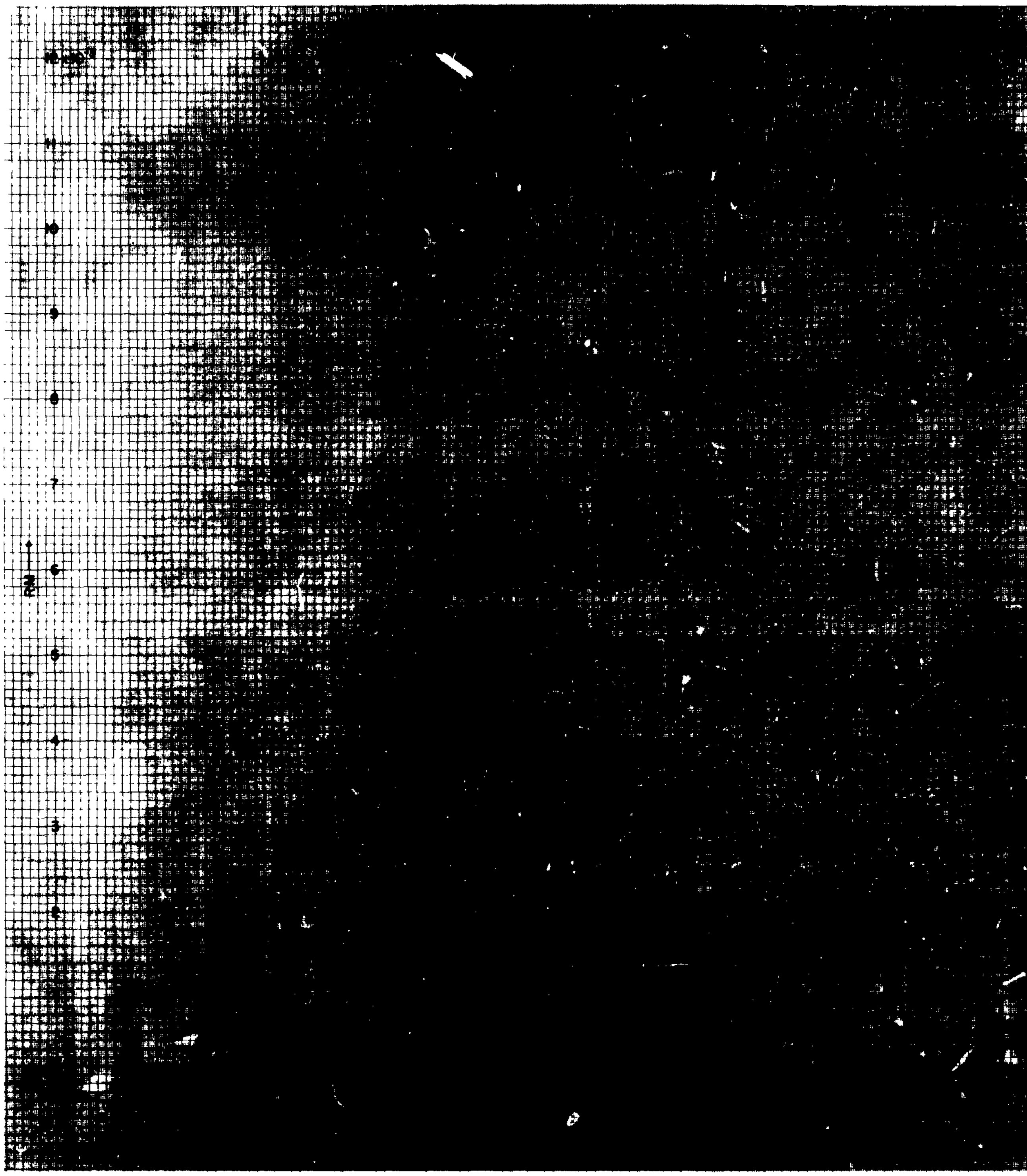
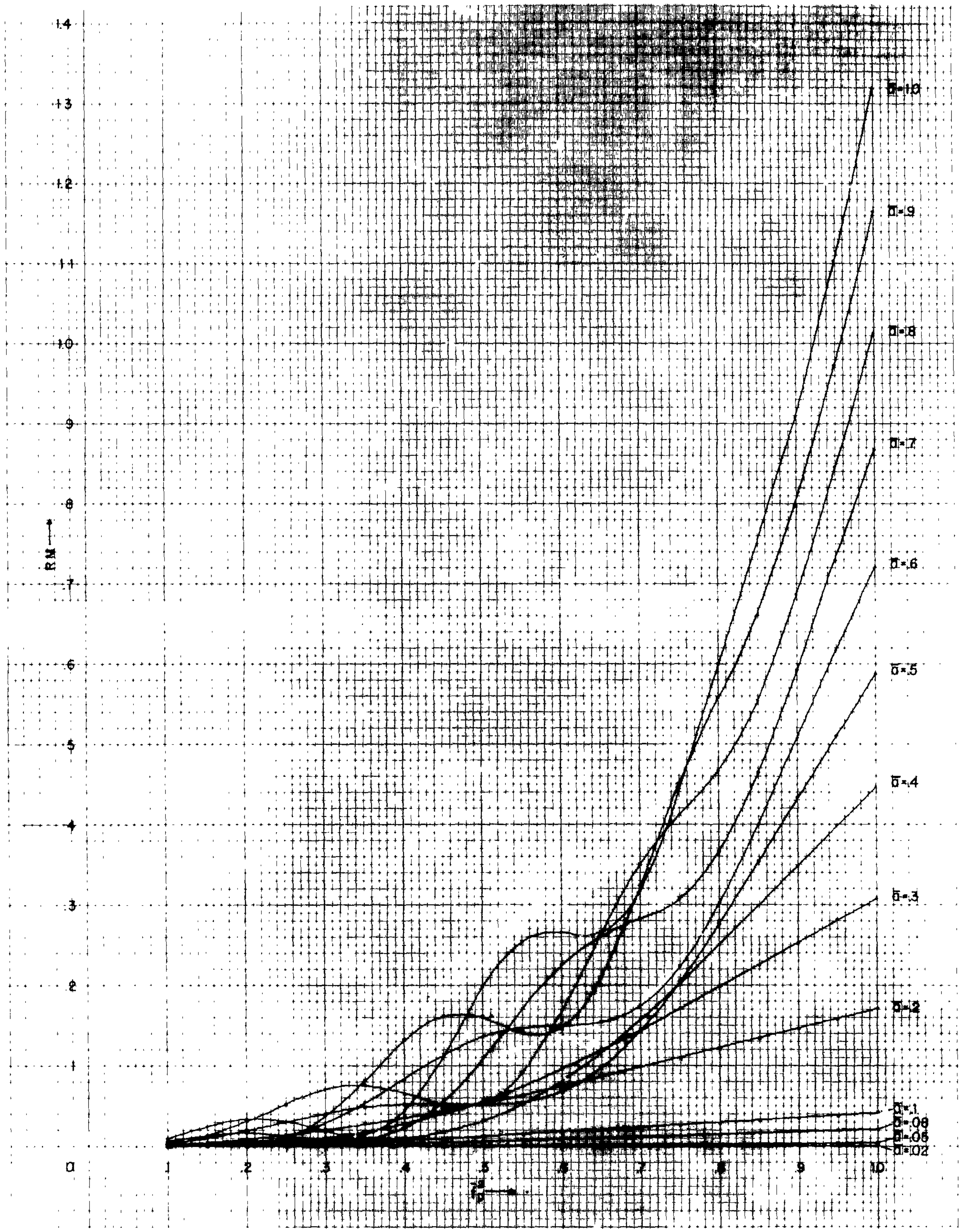


Figure 34



1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
RM
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
$\bar{\sigma}$=1.0
$\bar{\sigma}$=.9
$\bar{\sigma}$=.8
$\bar{\sigma}$=.7
$\bar{\sigma}$=.6
$\bar{\sigma}$=.5
$\bar{\sigma}$=.4
$\bar{\sigma}$=.3
$\bar{\sigma}$=.2
$\bar{\sigma}$=.1
$\bar{\sigma}$=.08
$\bar{\sigma}$=.05
$\bar{\sigma}$=.02

Figure 35

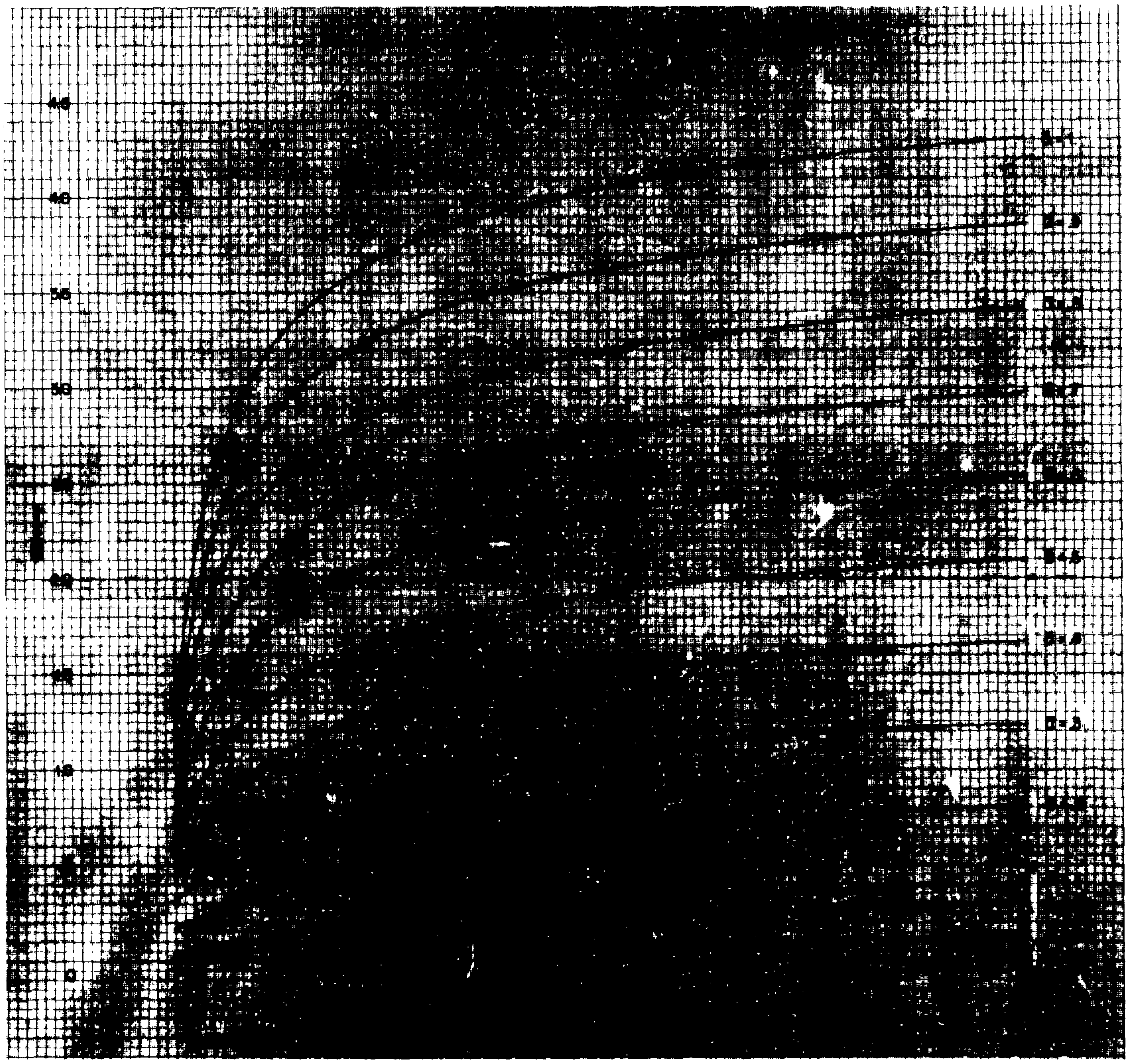


Figure 36

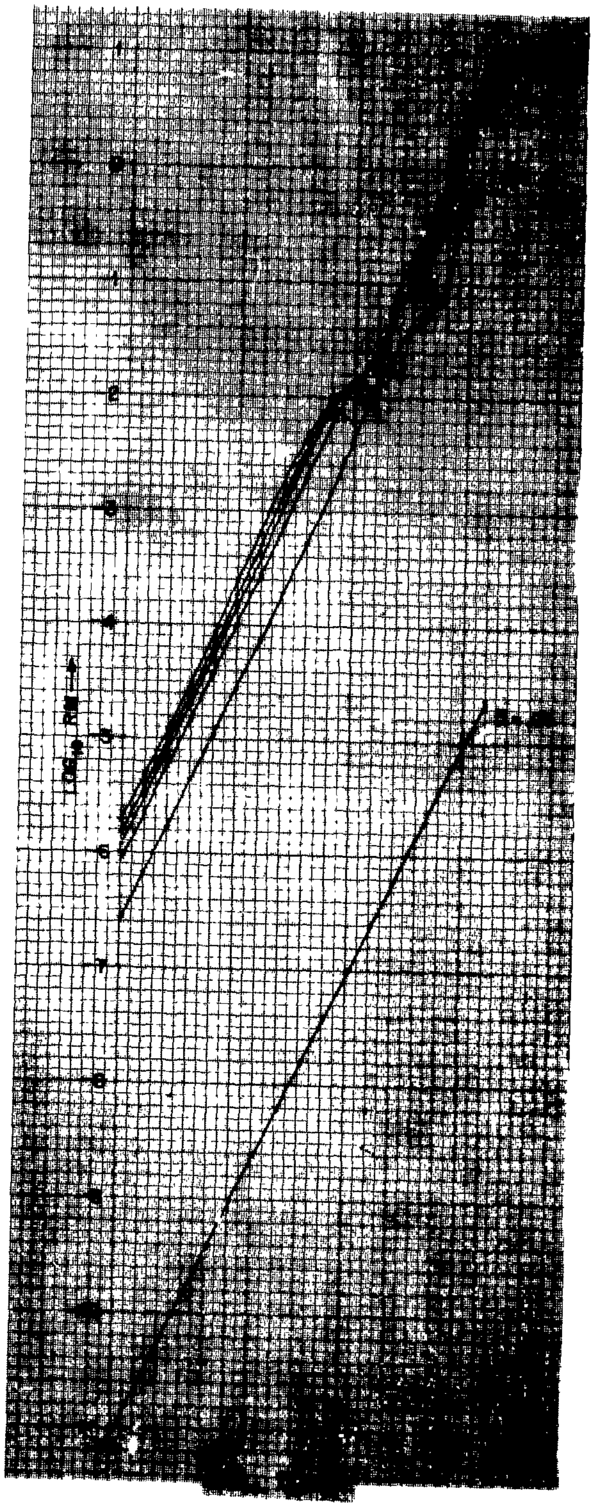


Figure 3(

THIS GRAPH SHOULD ONLY BE TAKEN QUALITATIVELY, THE RANGE IS BROKEN UP AND MORE DETAIL IS SHOWN IN SUCCEEDING GRAPHS.

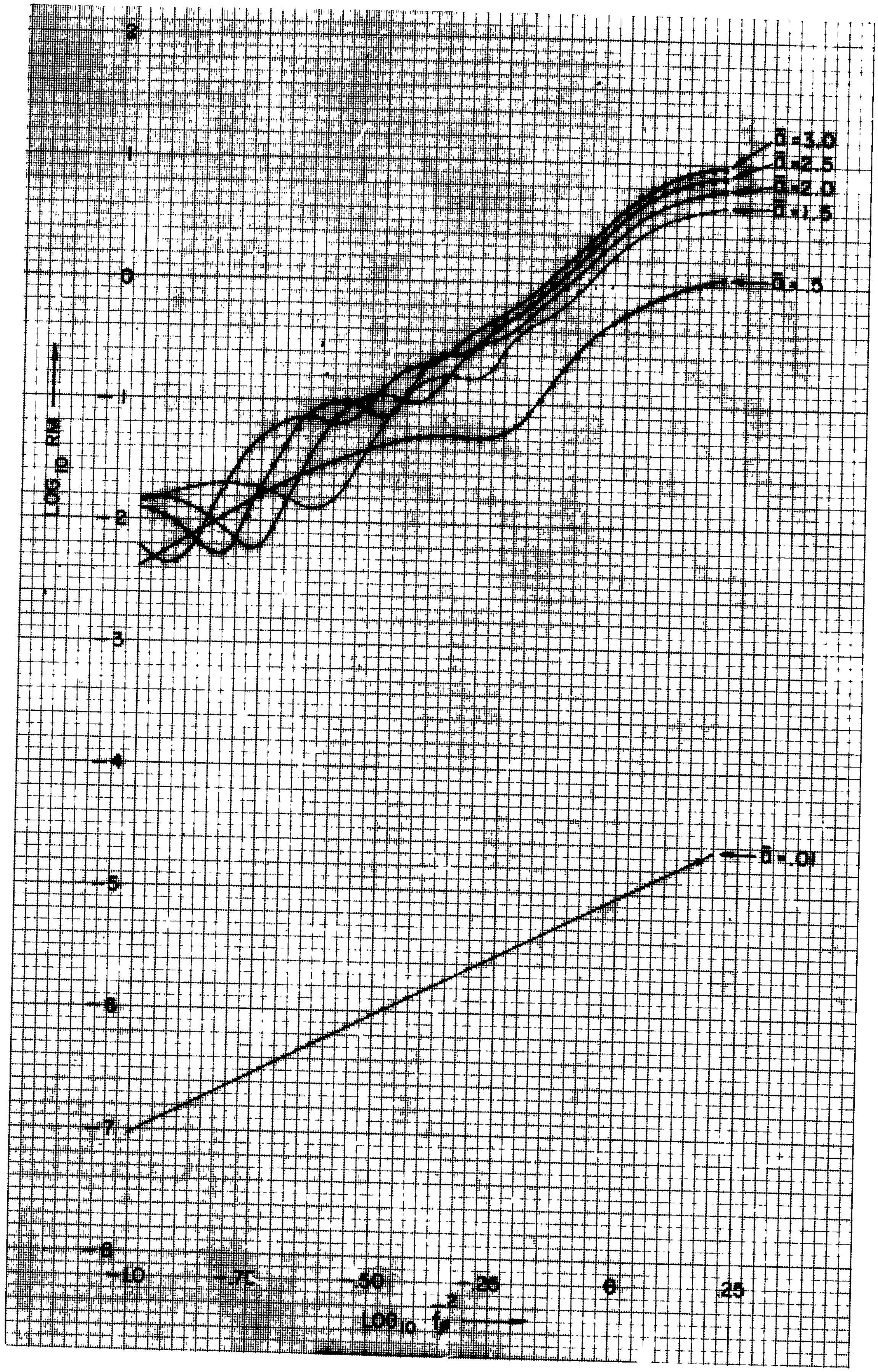


Figure 38

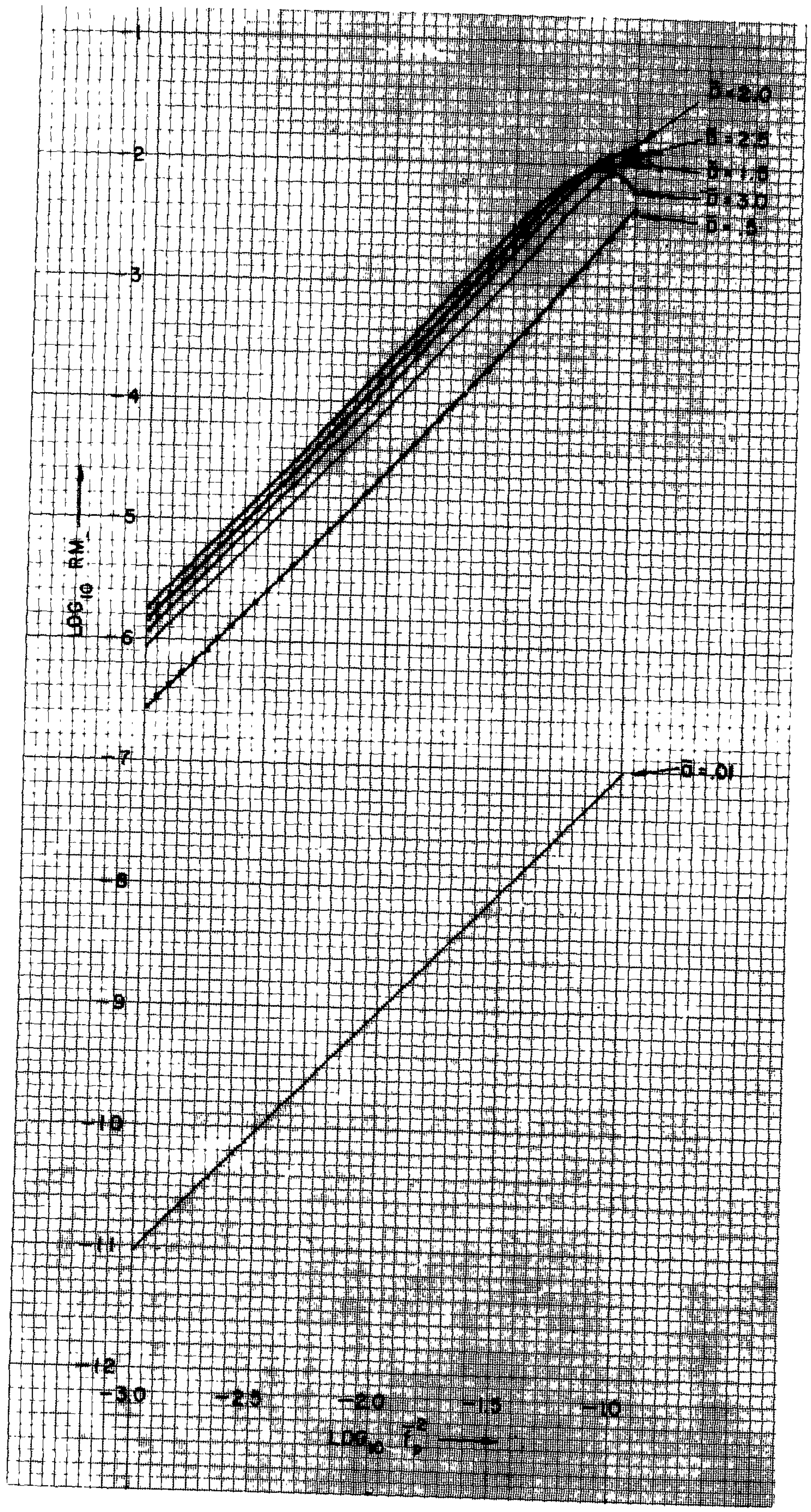
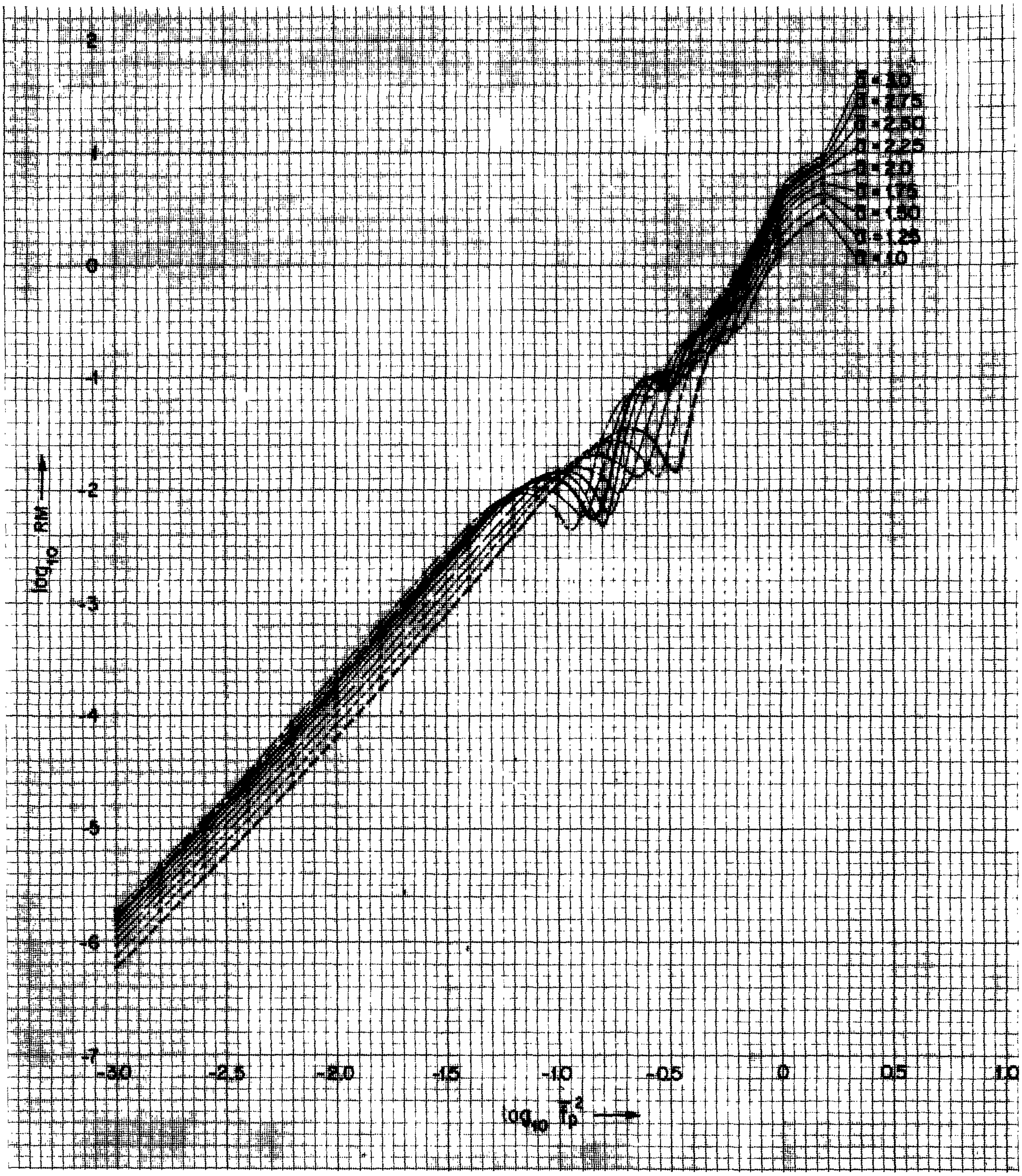
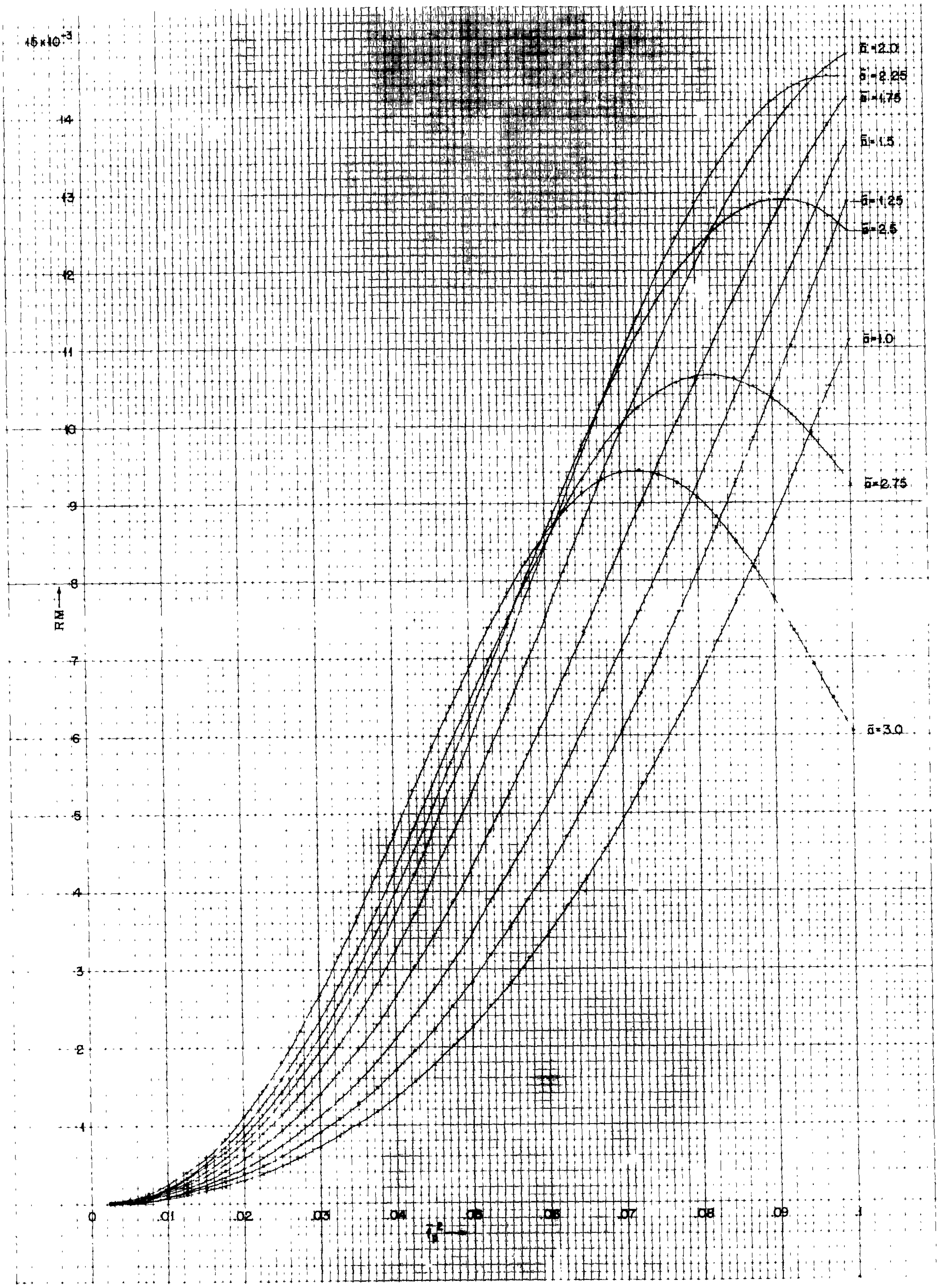


Figure 39



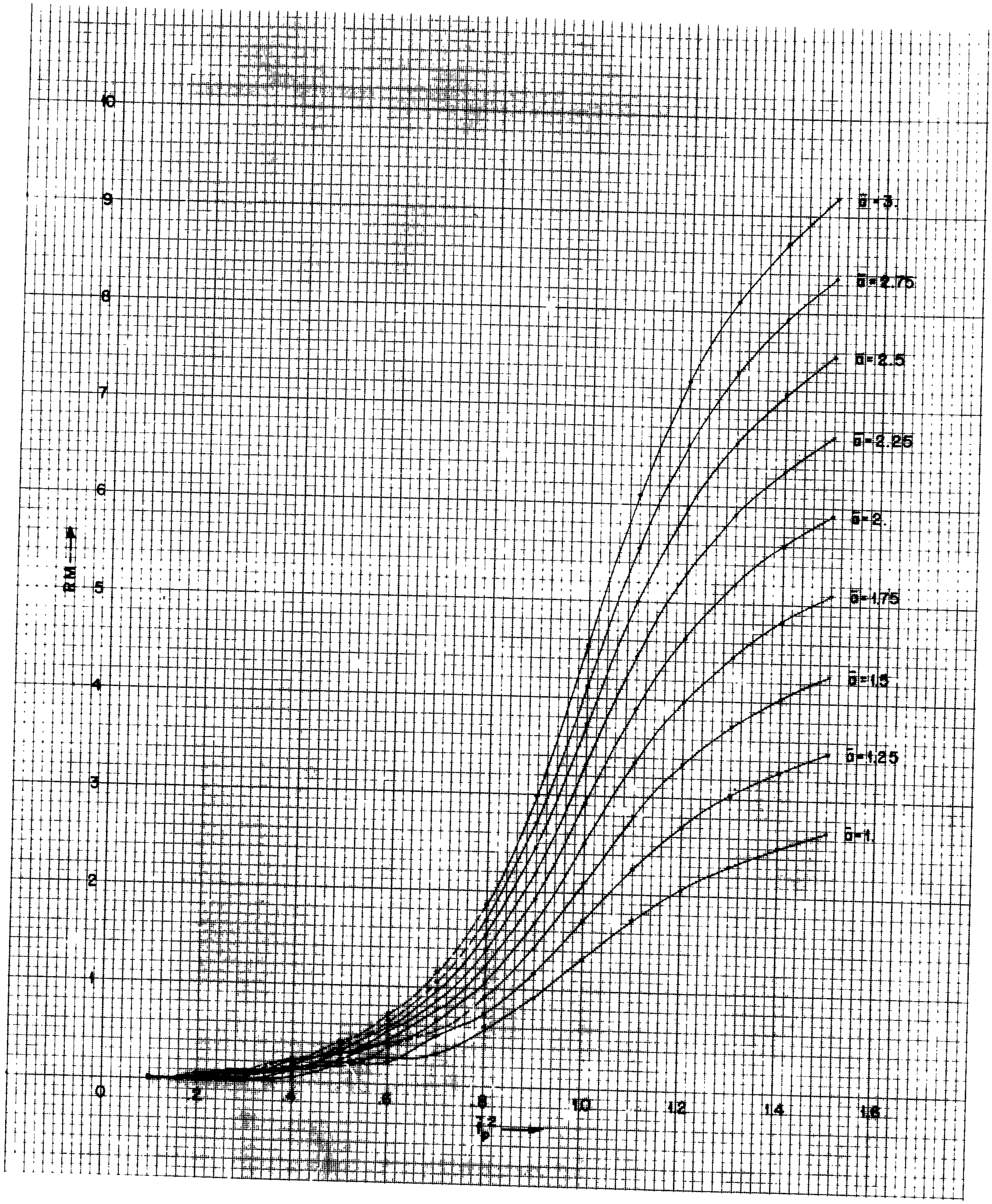
= 3.0
= 2.75
= 2.50
= 2.25
= 2.0
= 1.75
= 1.50
= 1.25
= 1.0
2
1
0
-1
-2
-3
-4
-5
-6
-7
log10 RM
-3.0
-2.5
-2.0
-1.5
-1.0
-0.5
0
0.5
1.0
log10 Tp2

Figure 40



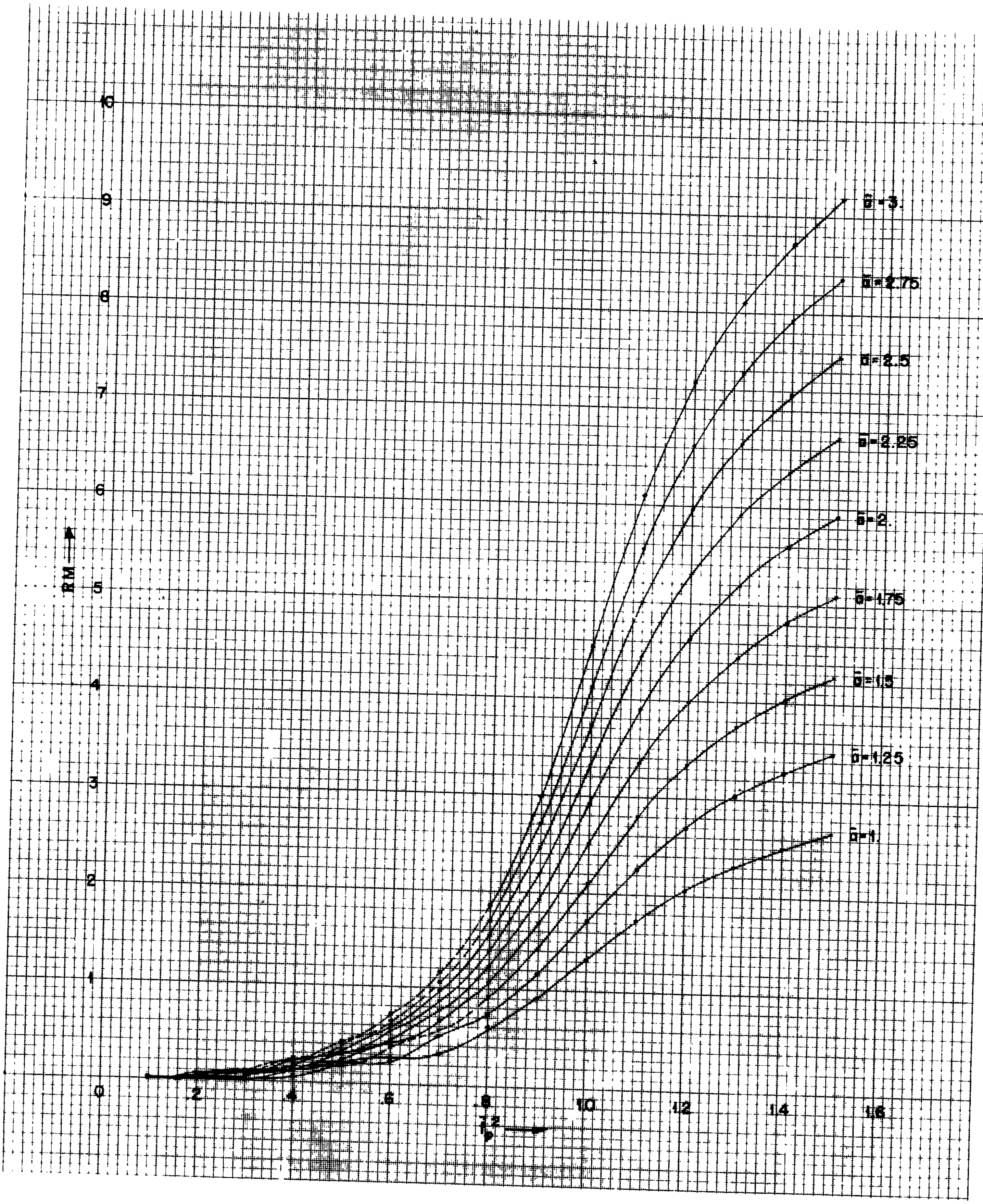
15×10⁻³
14
13
12
11
10
9
8
7
6
5
4
3
2
1
RM
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.1
$\bar{a}$=2.0
$\bar{a}$=2.25
$\bar{a}$=1.75
$\bar{a}$=1.5
$\bar{a}$=1.25
$\bar{a}$=2.5
$\bar{a}$=1.0
$\bar{a}$=2.75
$\bar{a}$=3.0

Figure 41



10
9
8
7
6
5
4
3
2
1
0
RM
.2
.4
.6
.8
1.0
1.2
1.4
1.6
$\bar{a}$=3.
$\bar{a}$=2.75
$\bar{a}$=2.5
$\bar{a}$=2.25
$\bar{a}$=2.
$\bar{a}$=1.75
$\bar{a}$=1.5
$\bar{a}$=1.25
$\bar{a}$=1.

Figure 42



RM
$\bar{T}_p^2$
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1
2
3
4
5
6
7
8
9
10
$\bar{a}=3.$
$\bar{a}=2.75$
$\bar{a}=2.5$
$\bar{a}=2.25$
$\bar{a}=2.$
$\bar{a}=1.75$
$\bar{a}=1.5$
$\bar{a}=1.25$
$\bar{a}=1.$

Figure 42

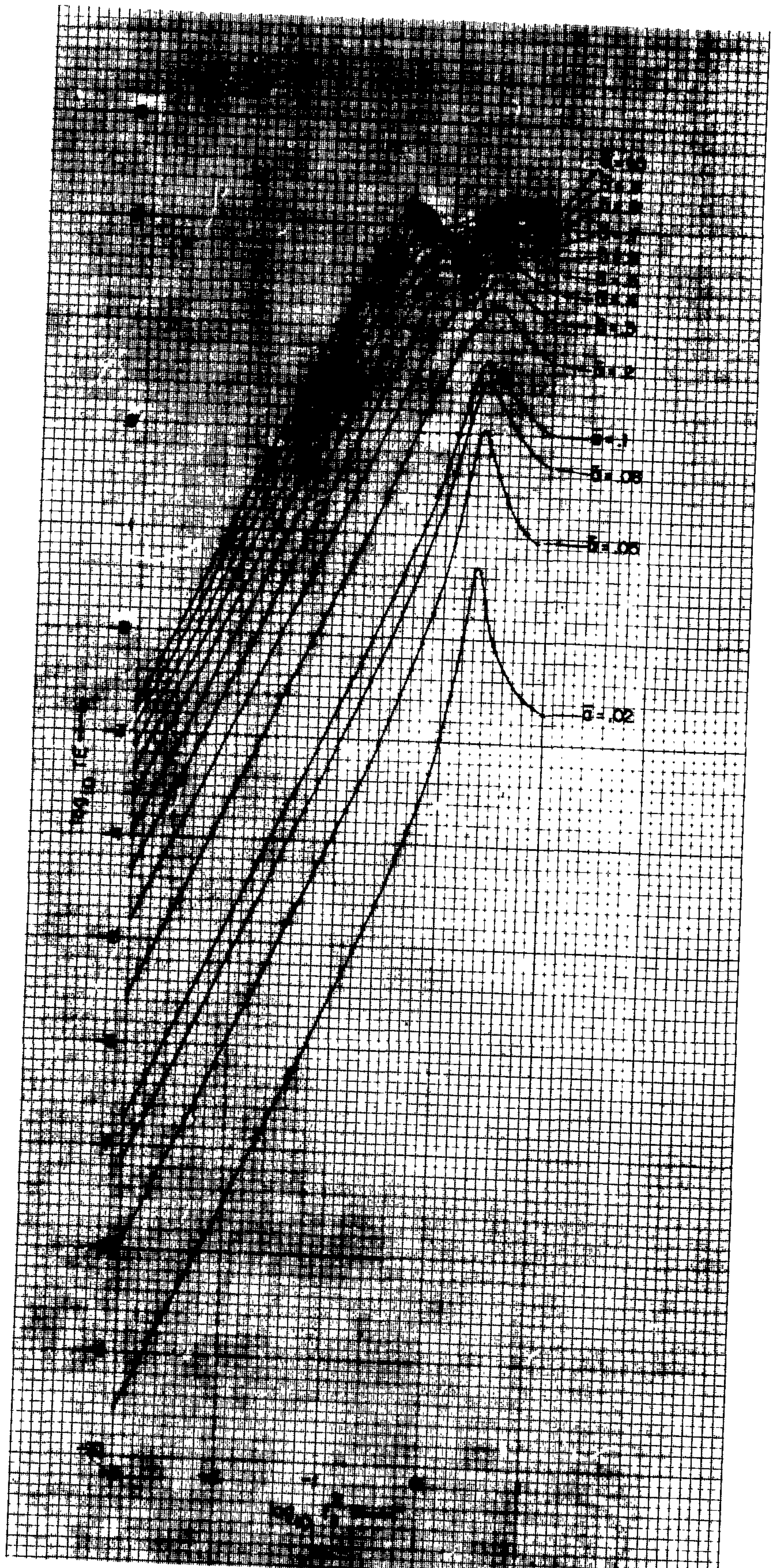


Figure 43

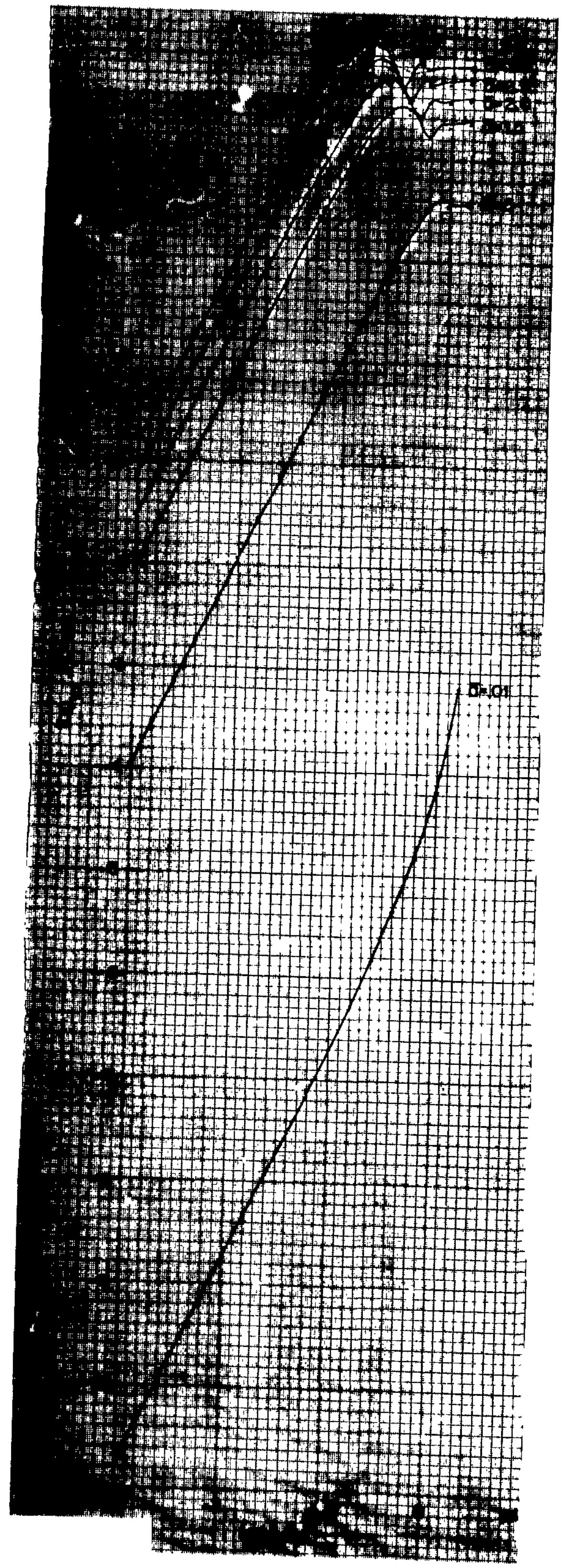


Figure 44

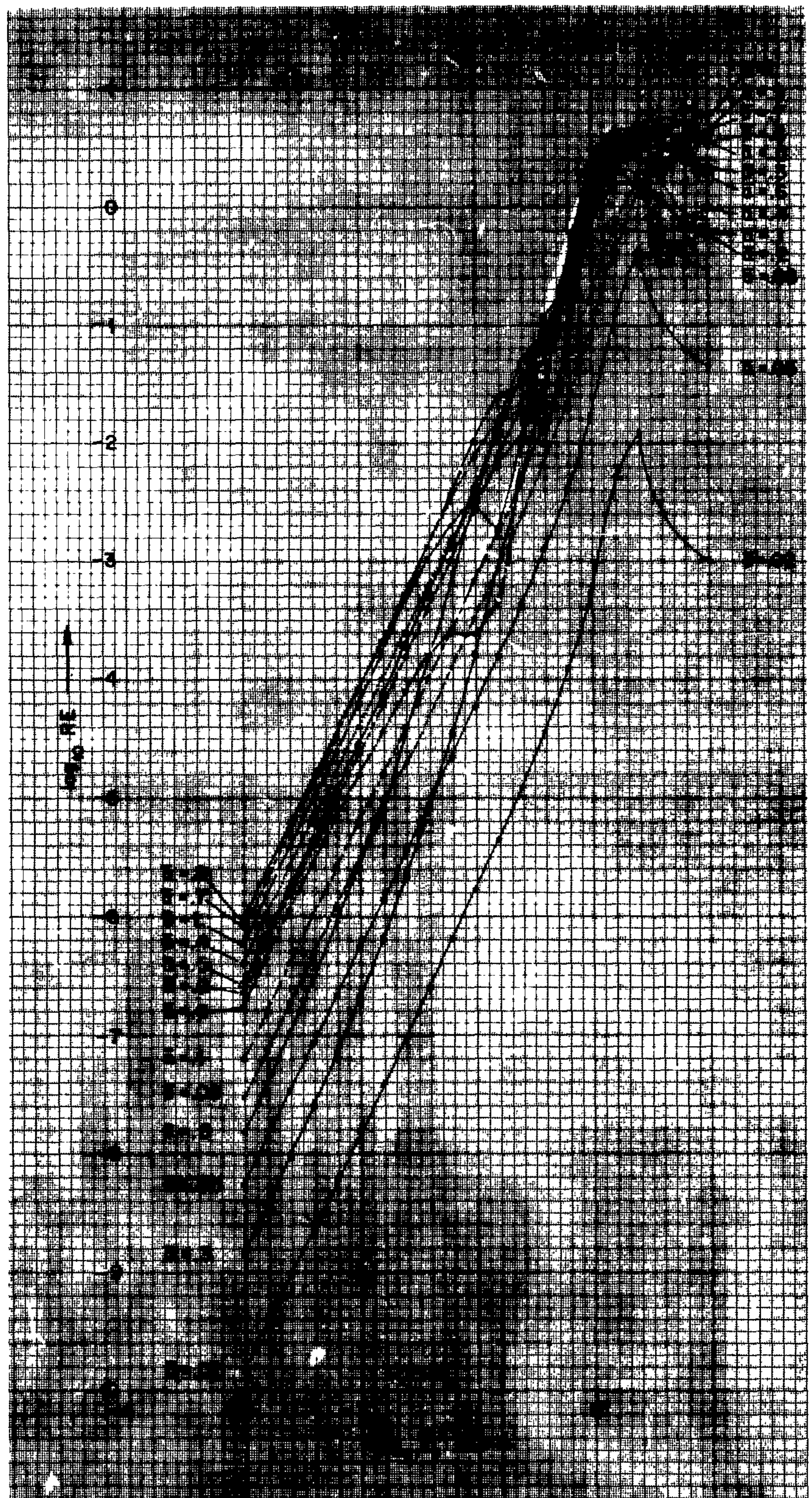


Figure 45

THIS GRAPH SHOULD ONLY BE TAKEN QUALITATIVELY,
THE RANGE IS BROKEN UP AND MORE DETAIL IS
SHOWN IN SUCCEEDING GRAPHS.

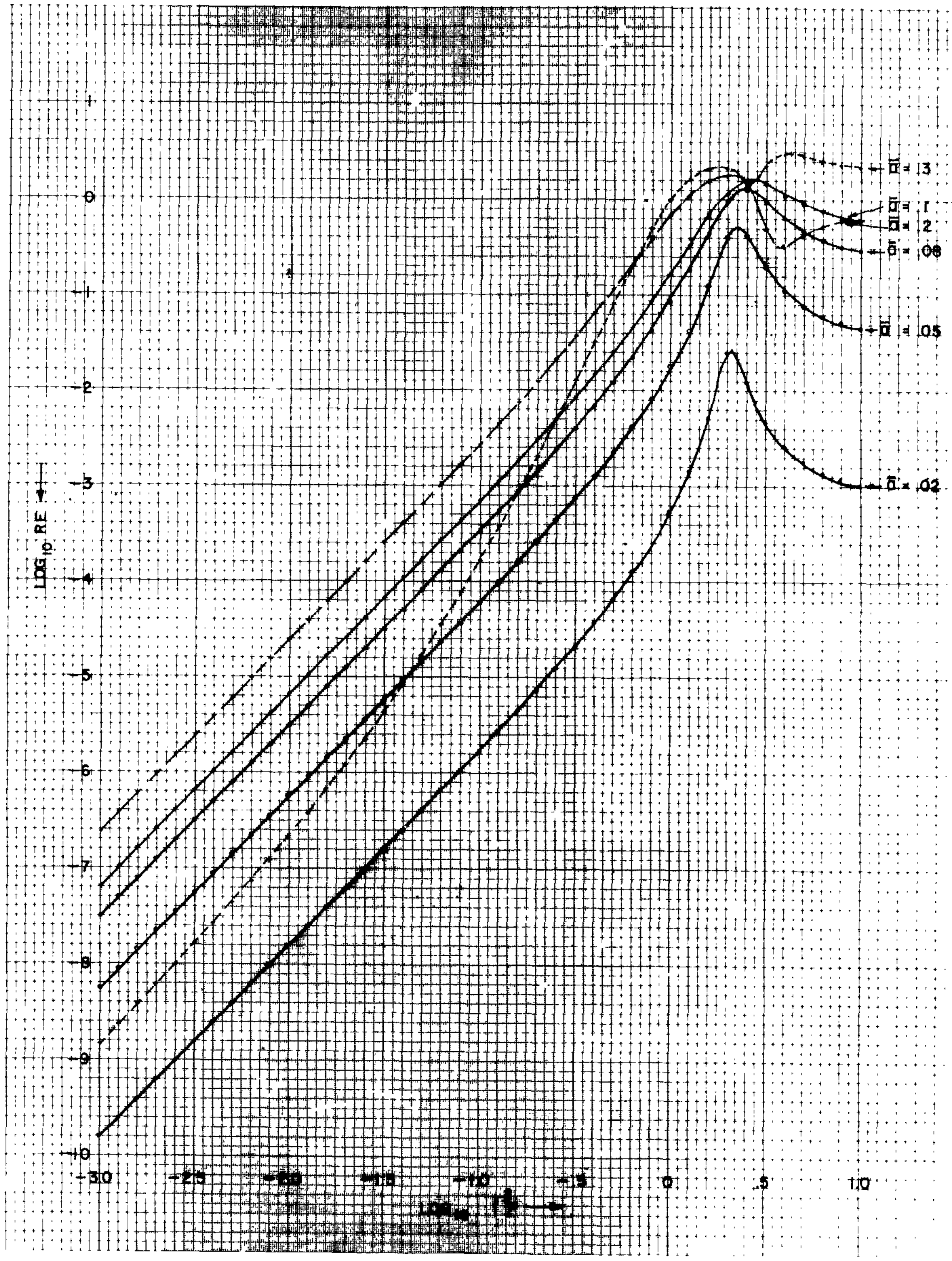


Figure 46

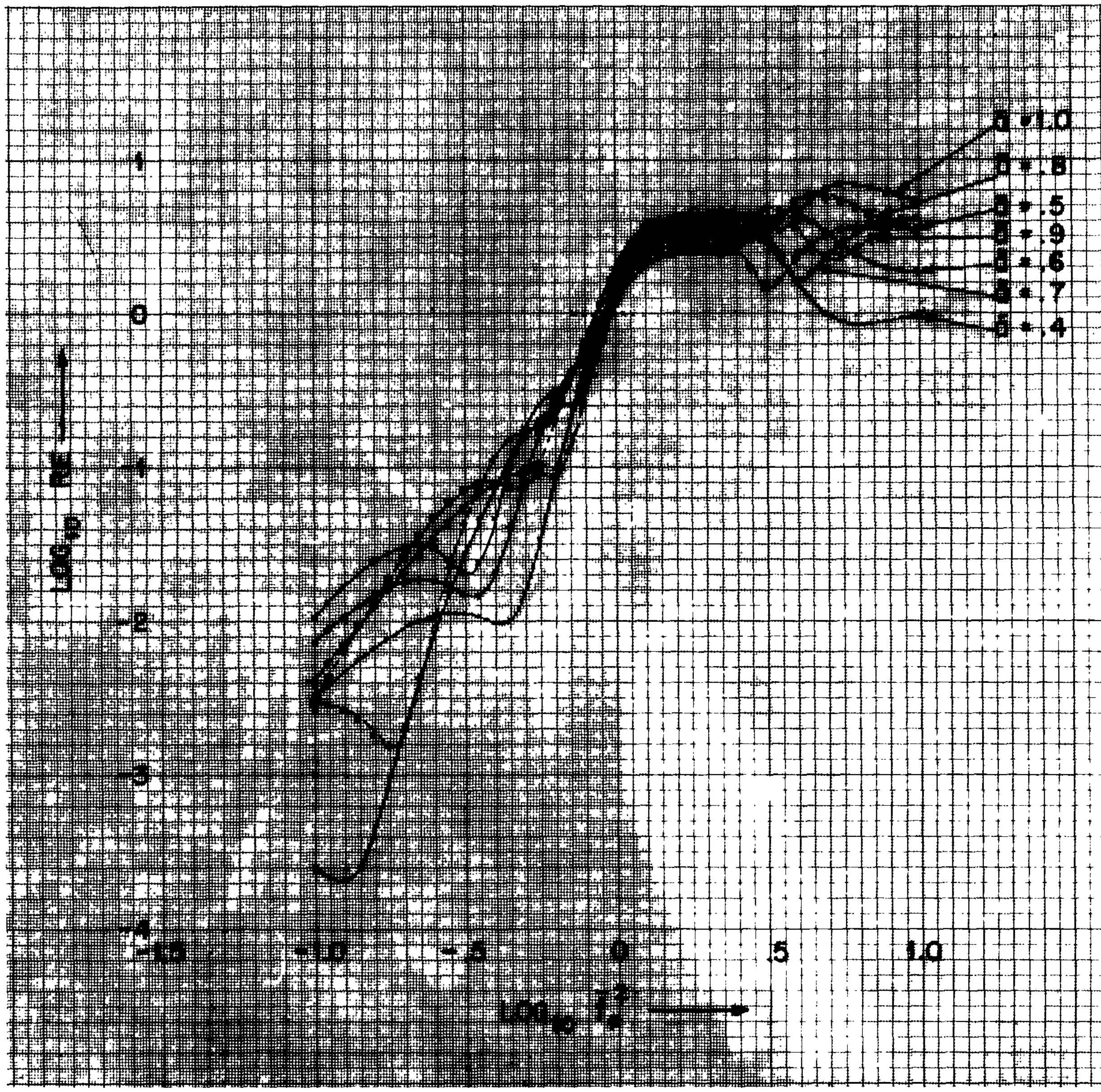


Figure 47

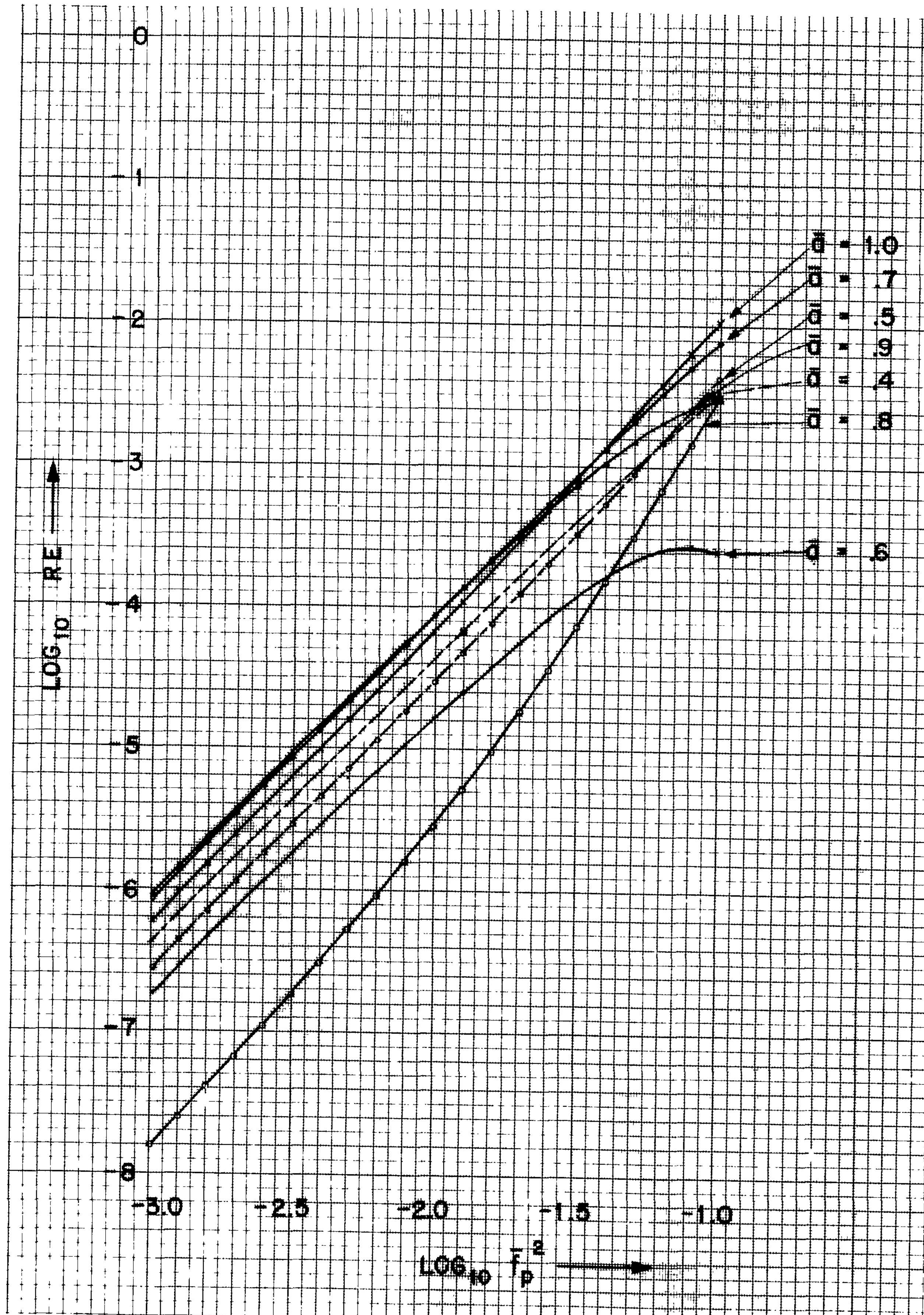

0
-1
-2
-3
-4
-5
-6
-7
-8
LOG$_{10}$ R.E.
-3.0
-2.5
-2.0
-1.5
-1.0
LOG$_{10}$ $\bar{f}_p^2$
d = 1.0
d = .7
d = .5
d = .9
d = .4
d = .8
d = .6

Figure 48

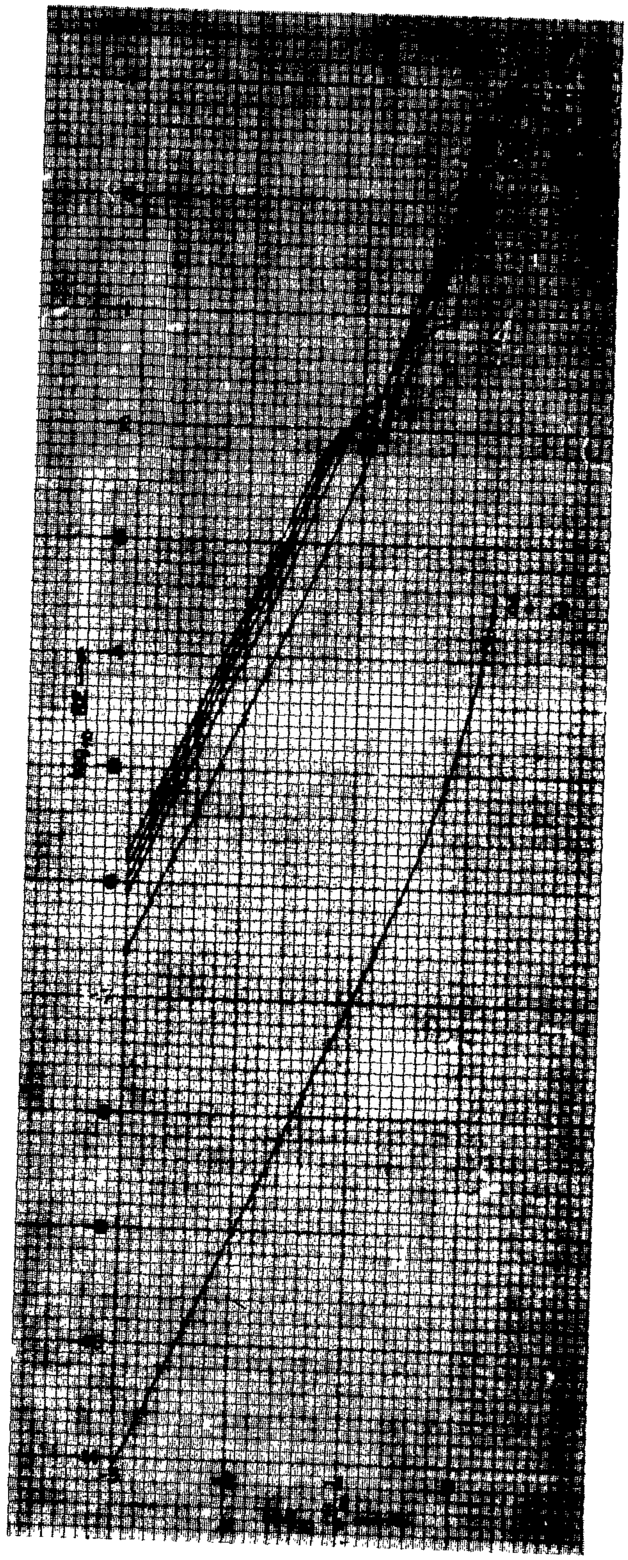


Figure 4?

THIS GRAPH SHOULD ONLY BE TAKI.N QUALITATIVELY, THE RANGE IS BROKEN UP AND MORE DETAIL IS SHOWN IN SUCCEEDING GRAPHS.

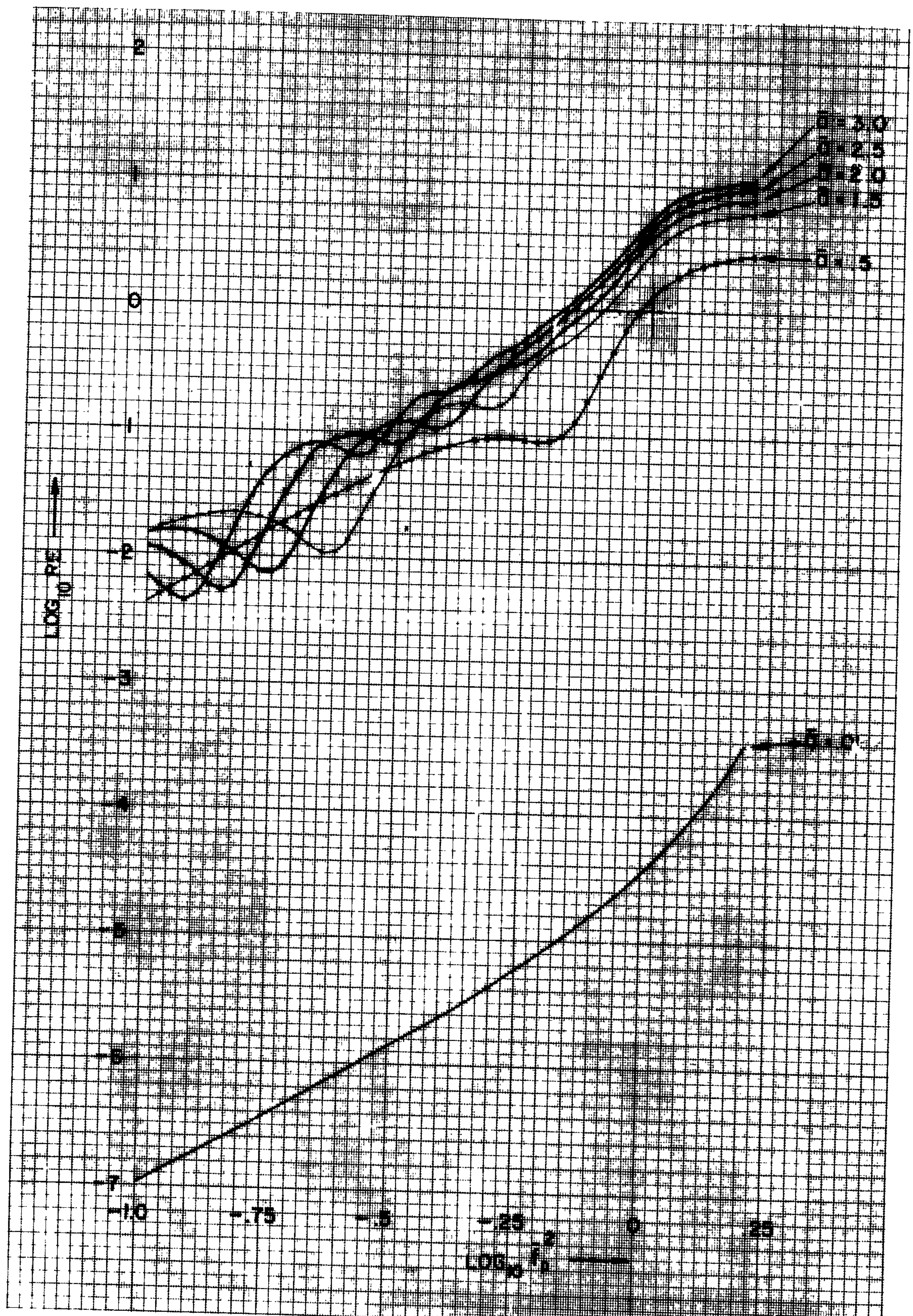
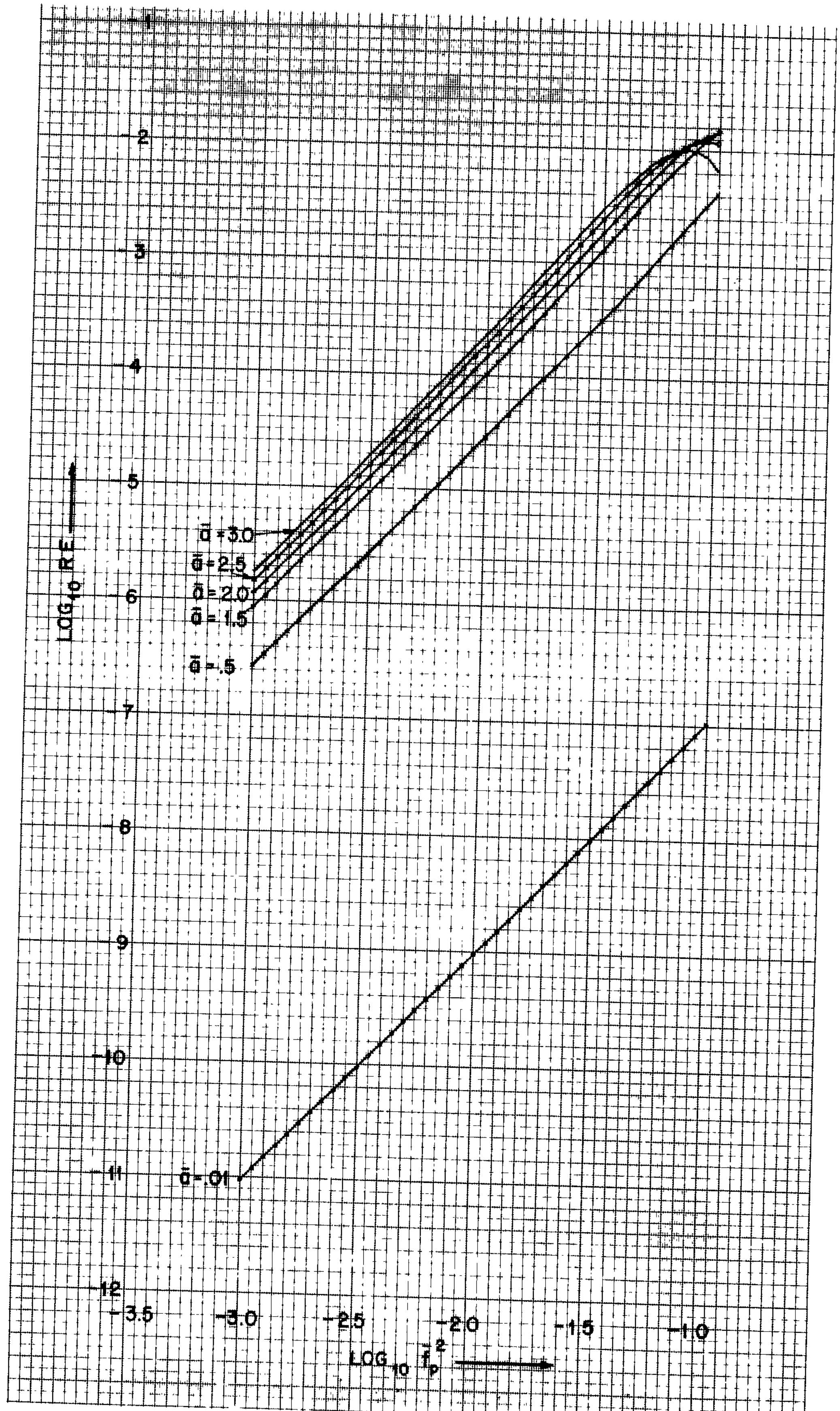
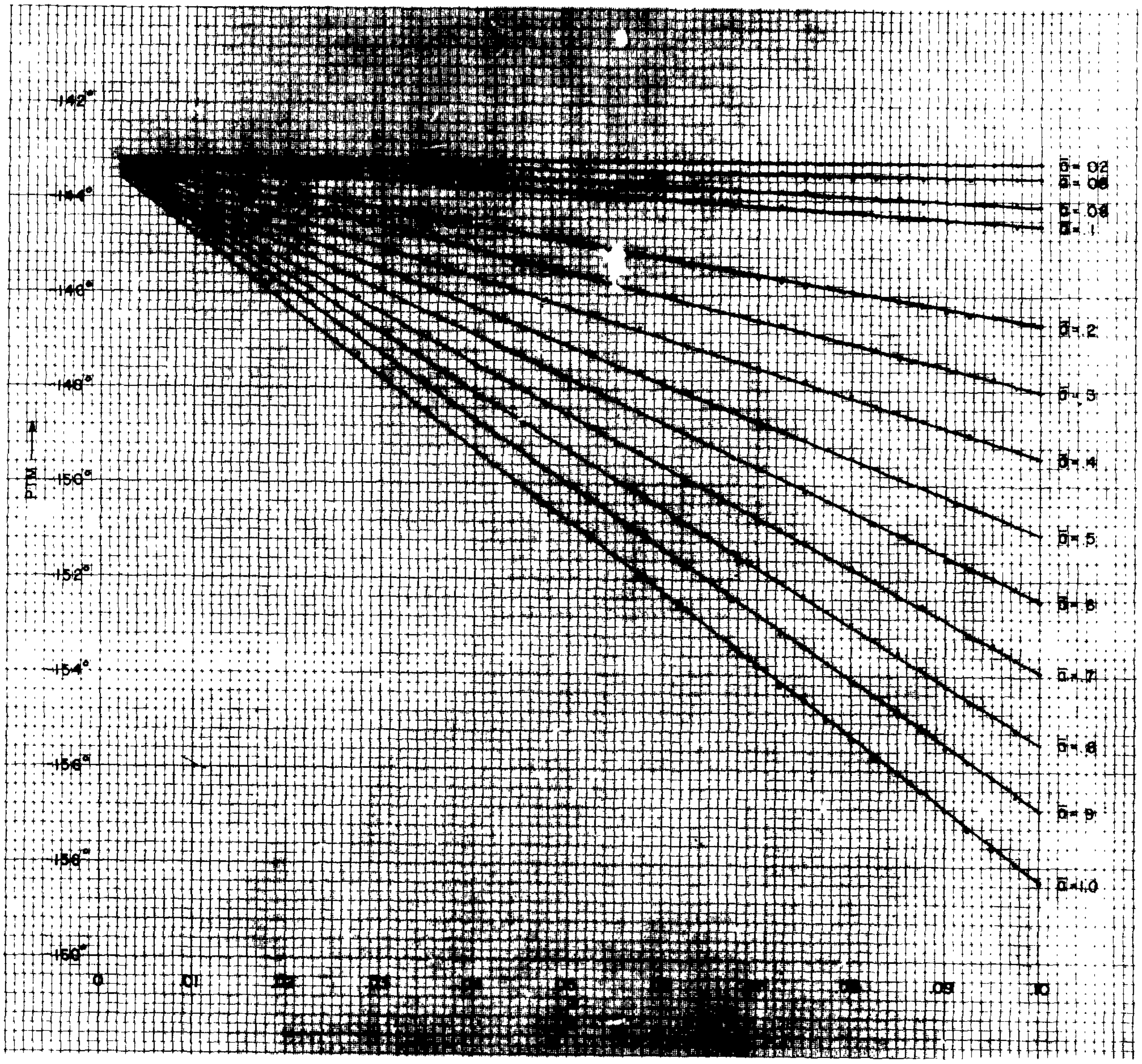


Figure 50



-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-11
-12
LOG10 RE
ā = 3.0
ā = 2.5
ā = 2.0
ā = 1.5
ā = .5
ā = .01
-3.5
-3.0
-2.5
-2.0
-1.5
-1.0
LOG10 f̄p²

Figure 51



PTM
142°
144°
146°
148°
150°
152°
154°
156°
158°
0
.01
.10
$\bar{\sigma}$ = .02
$\bar{\sigma}$ = .2
$\bar{\sigma}$ = .4
$\bar{\sigma}$ = .5
$\bar{\sigma}$ = .7
$\bar{\sigma}$ = .9
$\bar{\sigma}$ = 1.0

Figure 52

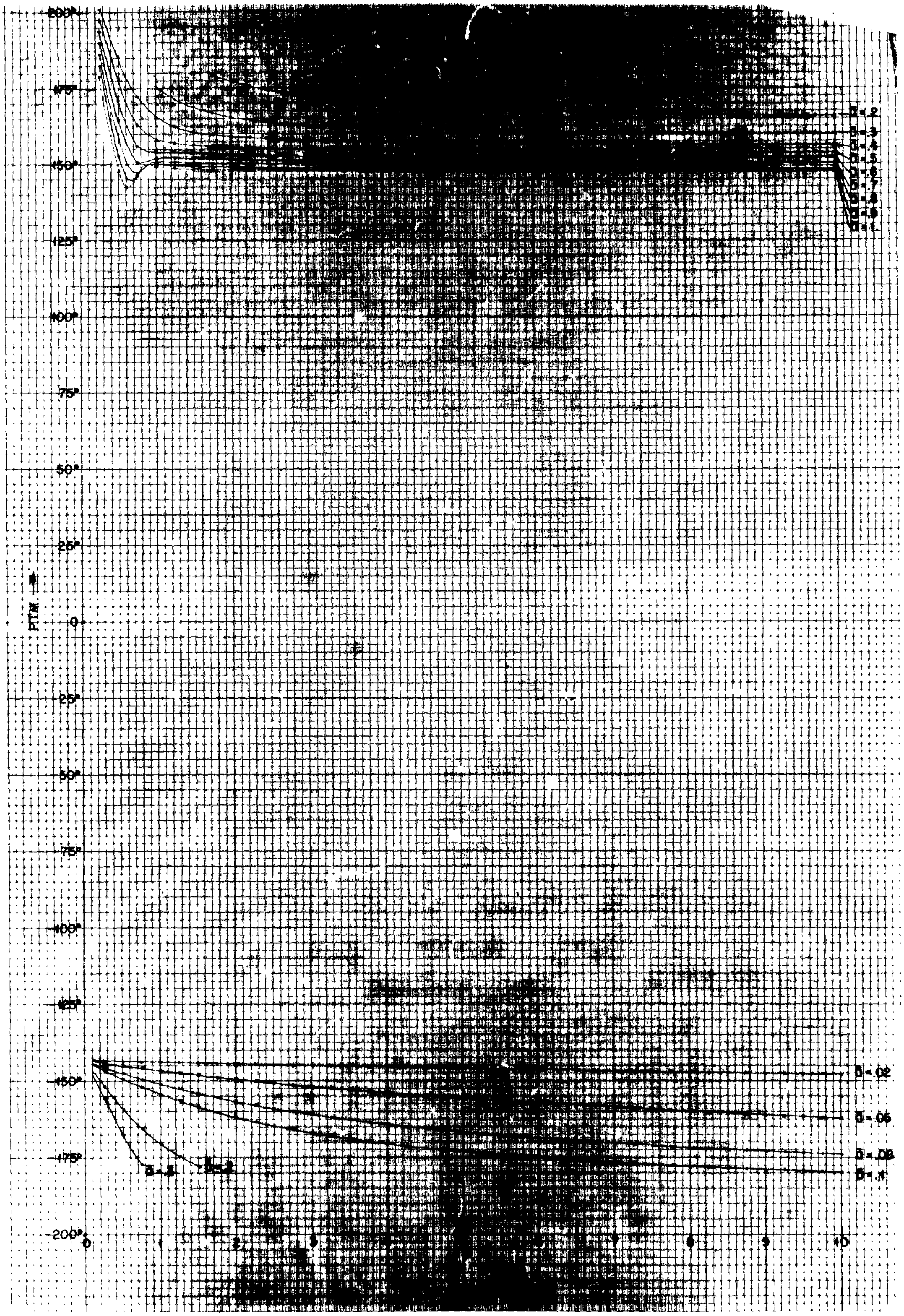

200°
175°
150°
125°
100°
75°
50°
25°
0
-25°
-50°
-75°
-100°
-125°
-150°
-175°
-200°
PTM
0
1
2
3
4
5
6
7
8
9
10
δ = .2
δ = .3
δ = .4
δ = .5
δ = .6
δ = .7
δ = .8
δ = .9
δ = 1.
δ = .02
δ = .06
δ = .08
δ = .1
δ = .5
δ = .2

Figure 53

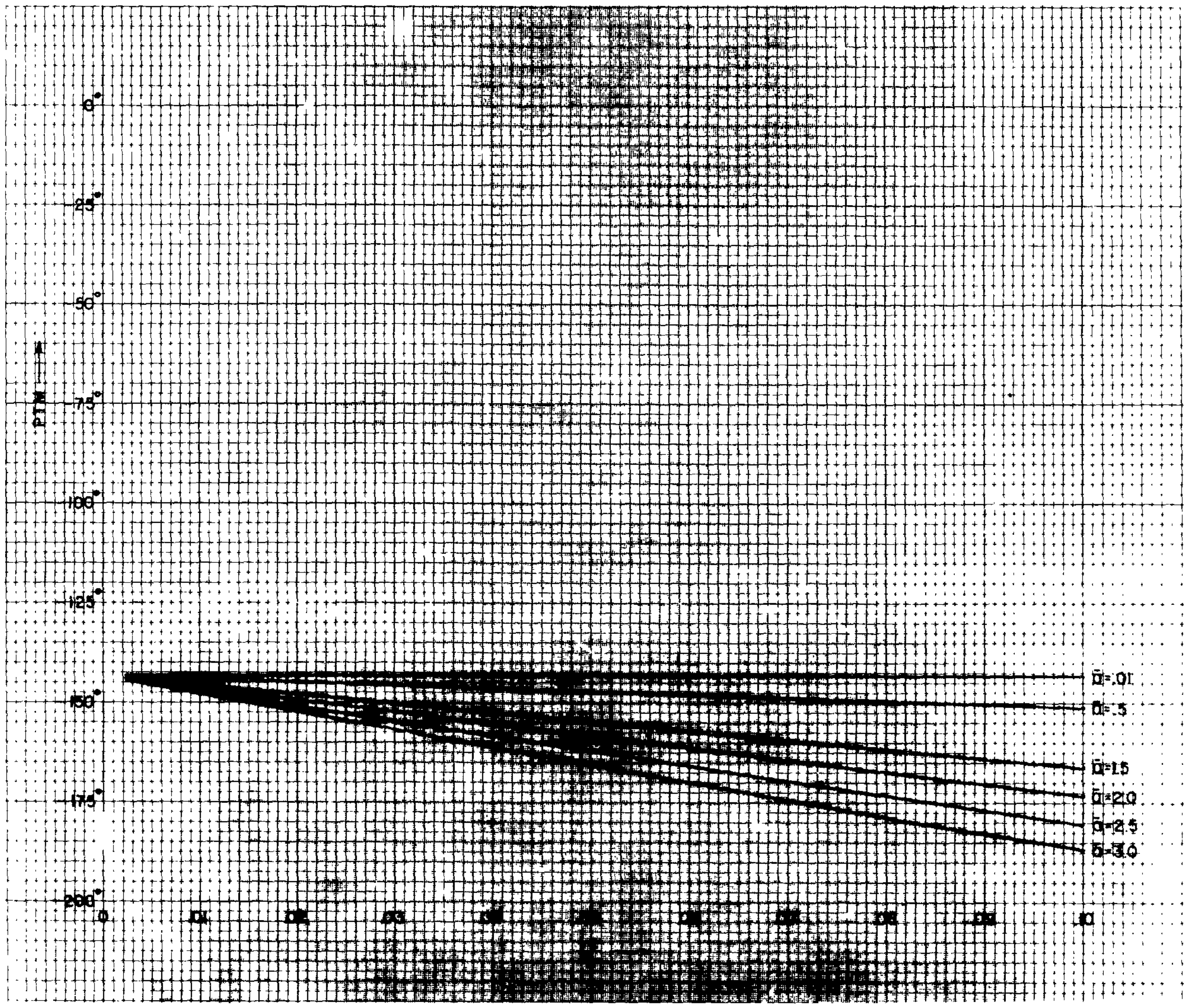
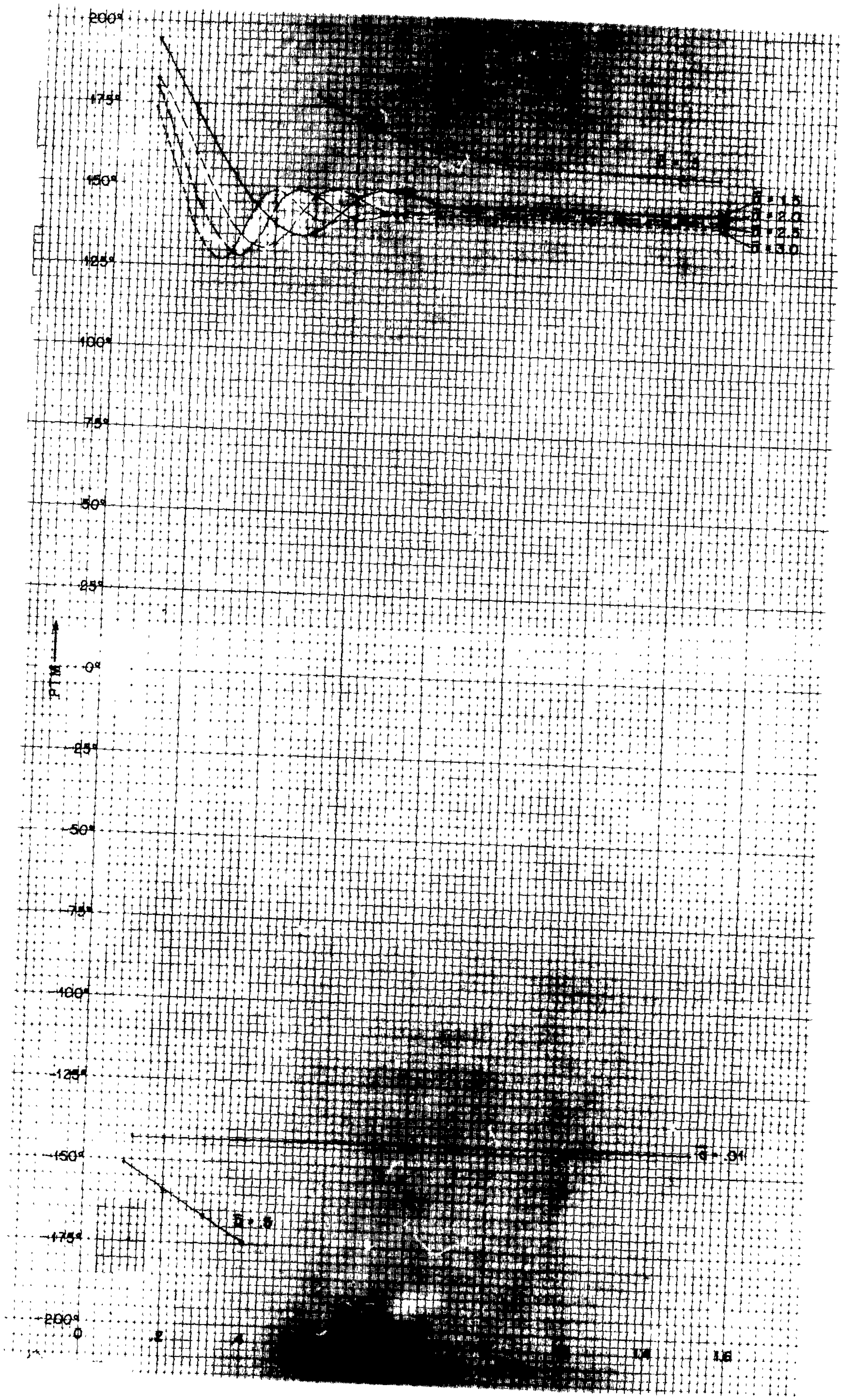
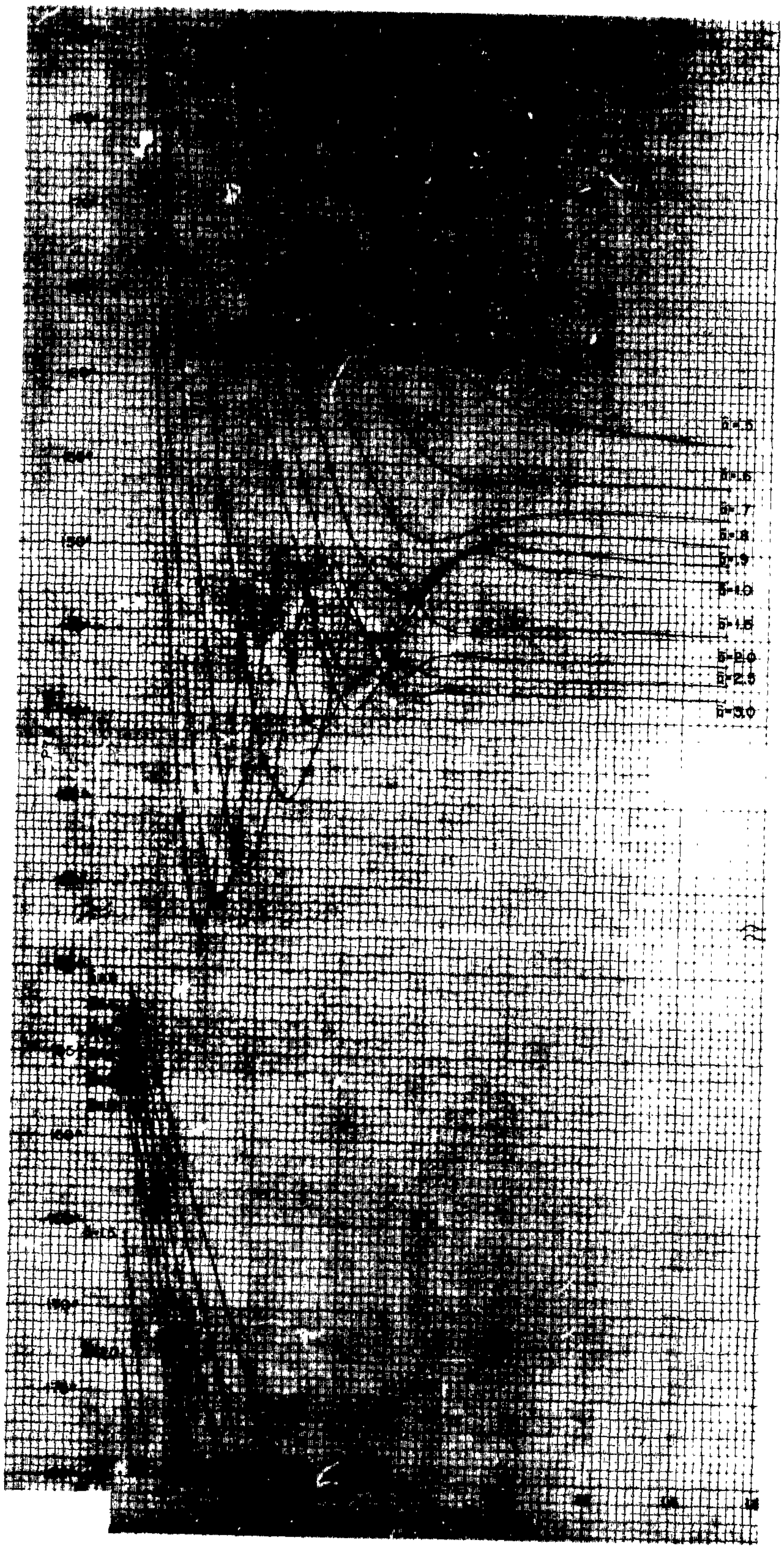


Figure 54



200°
175°
150°
125°
100°
75°
50°
25°
0°
-25°
-50°
-75°
-100°
-125°
-150°
-175°
-200°
PTM
D̄ = 1.5
D̄ = 2.0
D̄ = 2.5
D̄ = 3.0
D̄ = .01
D̄ = .5
0
.2
.4
1.4
1.6

Figure 55



$\bar{b}=.5$
$\bar{b}=.6$
$\bar{b}=.7$
$\bar{b}=.8$
$\bar{b}=.9$
$\bar{b}=1.0$
$\bar{b}=1.5$
$\bar{b}=2.0$
$\bar{b}=2.5$
$\bar{b}=3.0$

Figure 56

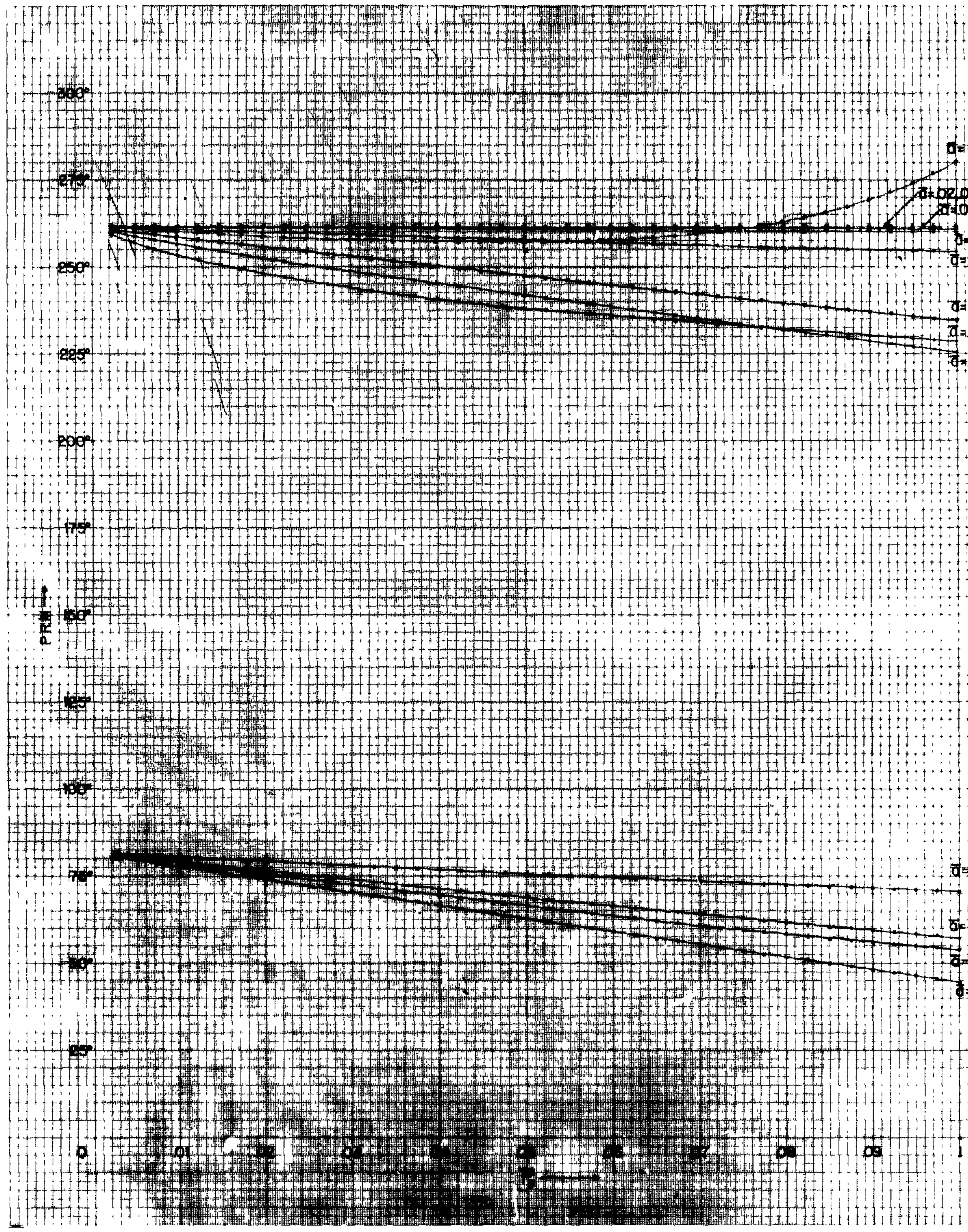


Figure 57

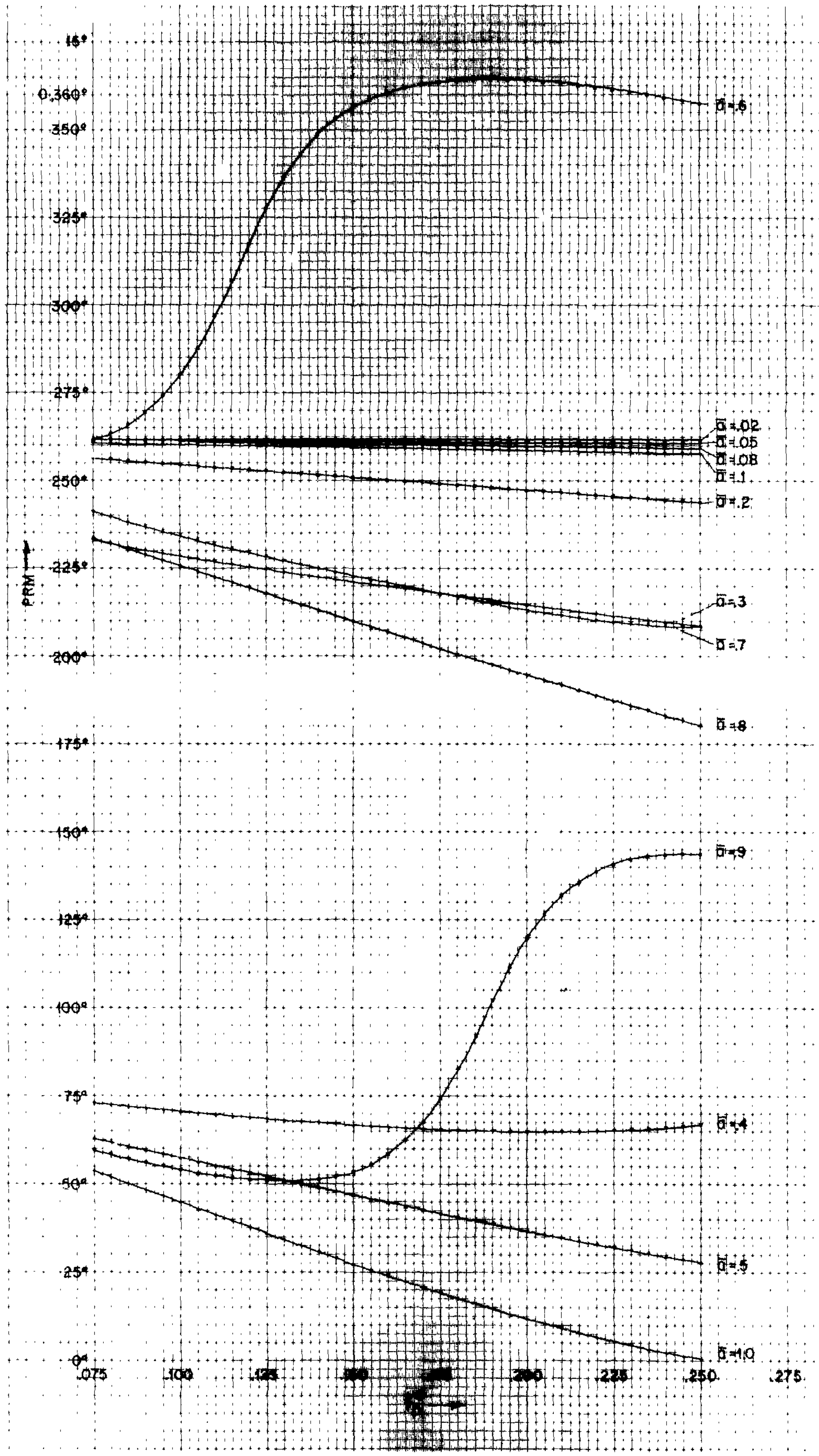
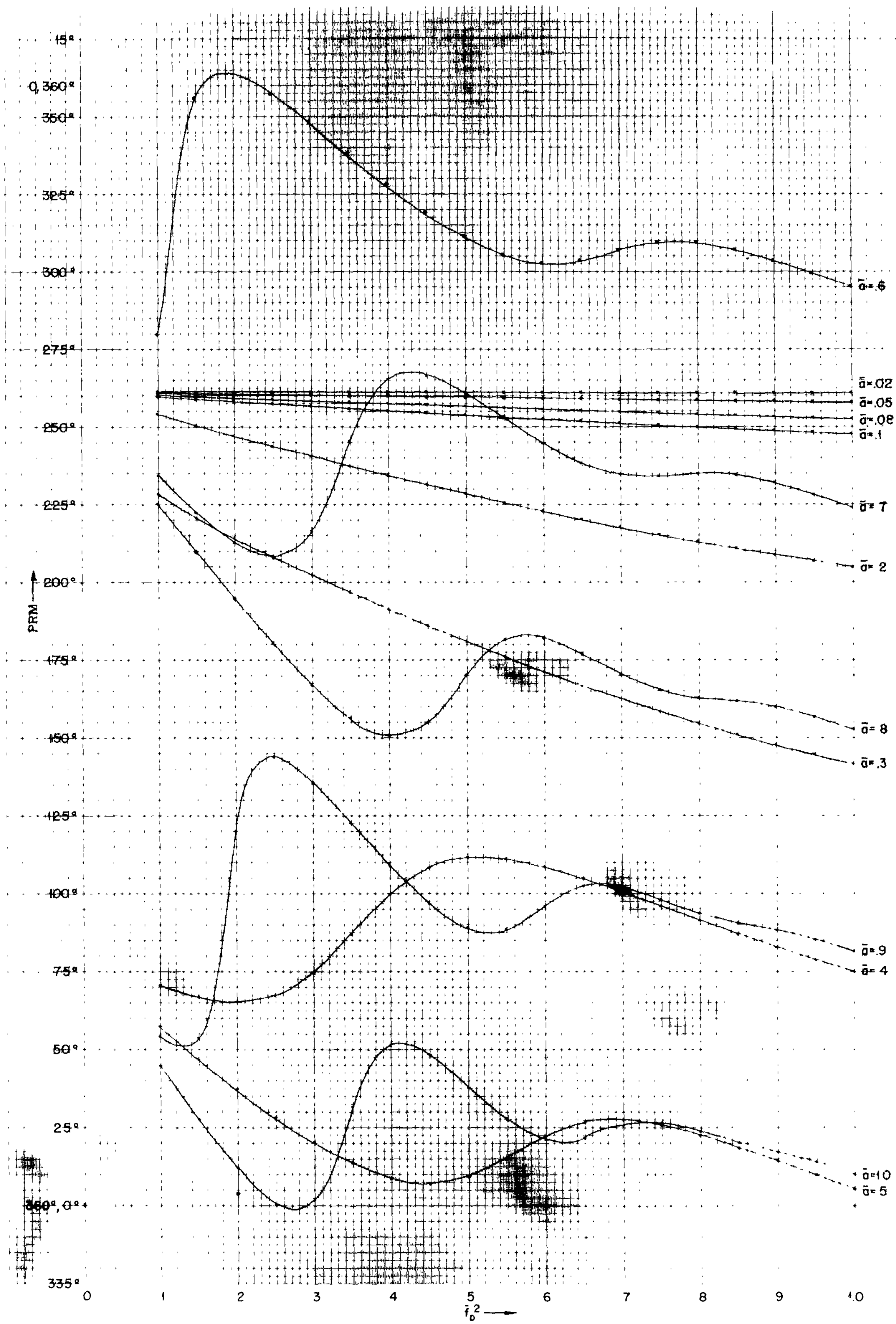
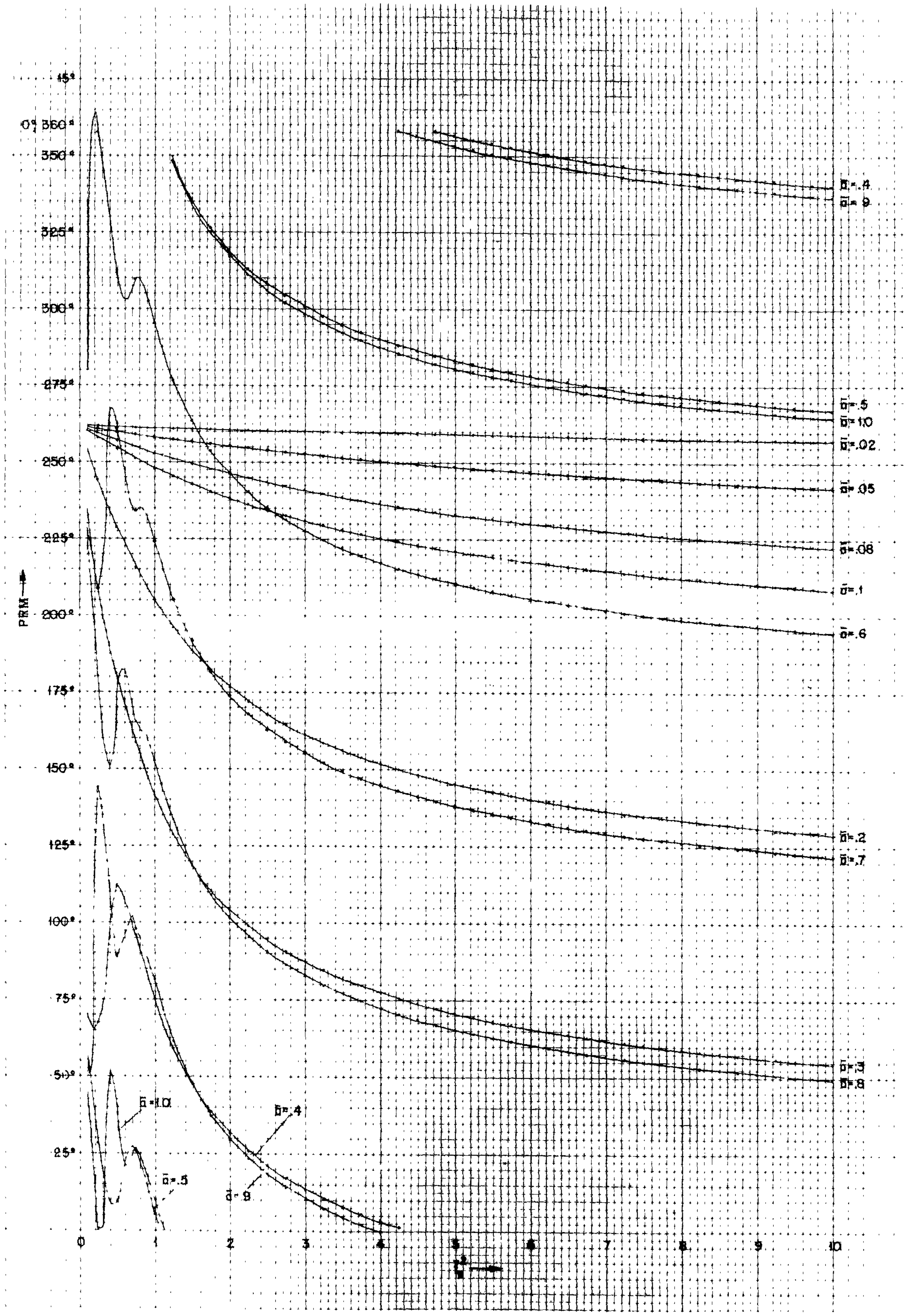

15°
0,360°
350°
325°
300°
275°
250°
225°
200°
175°
150°
125°
100°
75°
50°
25°
0°
PRM
.075
.100
.125
.200
.225
.250
.275
δ̄ =.6
δ̄ =.02
δ̄ =.05
δ̄ =.08
δ̄ =.1
δ̄ =.2
δ̄ =.3
δ̄ =.7
δ̄ =.8
δ̄ =.9
δ̄ =.4
δ̄ =.5
δ̄ =1.0

Figure 58



15°
0, 360°
350°
325°
300°
275°
250°
225°
200°
PRM
175°
150°
125°
100°
75°
50°
25°
360°, 0°
335°
0
1
2
3
4
5
6
7
8
9
10
$\bar{a}$=.6
$\bar{a}$=.02
$\bar{a}$=.05
$\bar{a}$=.08
$\bar{a}$=.1
$\bar{a}$=.7
$\bar{a}$=.2
$\bar{a}$=.8
$\bar{a}$=.3
$\bar{a}$=.9
$\bar{a}$=.4
$\bar{a}$=10
$\bar{a}$=.5

Figure)



45°
0° 360°
350°
325°
300°
275°
250°
225°
200°
175°
150°
125°
100°
75°
50°
25°
PRM
0
1
2
3
4
5
6
7
8
9
10
b̄=.4
b̄=.9
b̄=.5
b̄=1.0
b̄=.02
b̄=.05
b̄=.08
b̄=.1
b̄=.6
b̄=.2
b̄=.7
b̄=.3
b̄=.8
b̄=1.0
b̄=.4
b̄=.5
b̄=.9

Figure 30

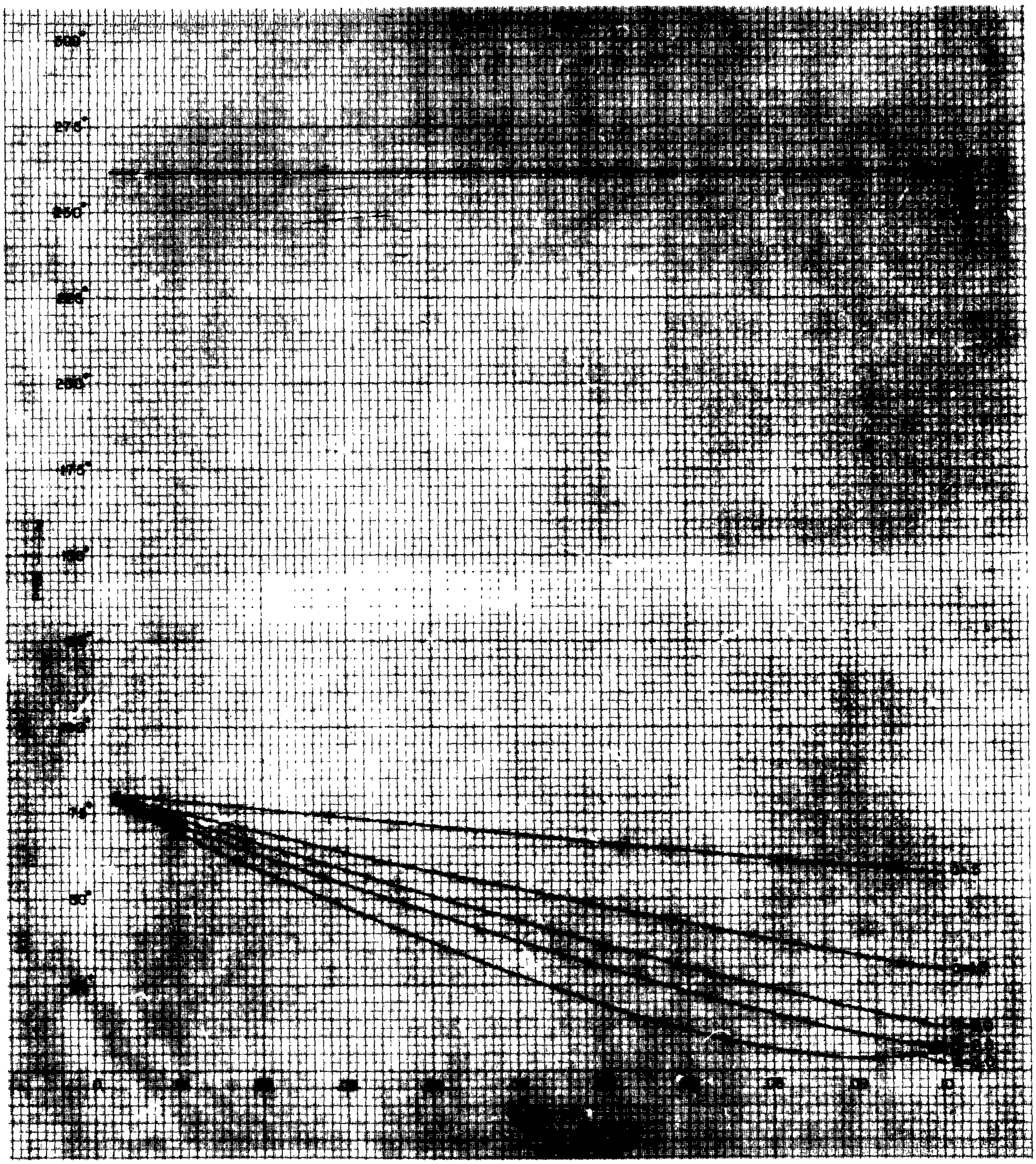


Figure 61

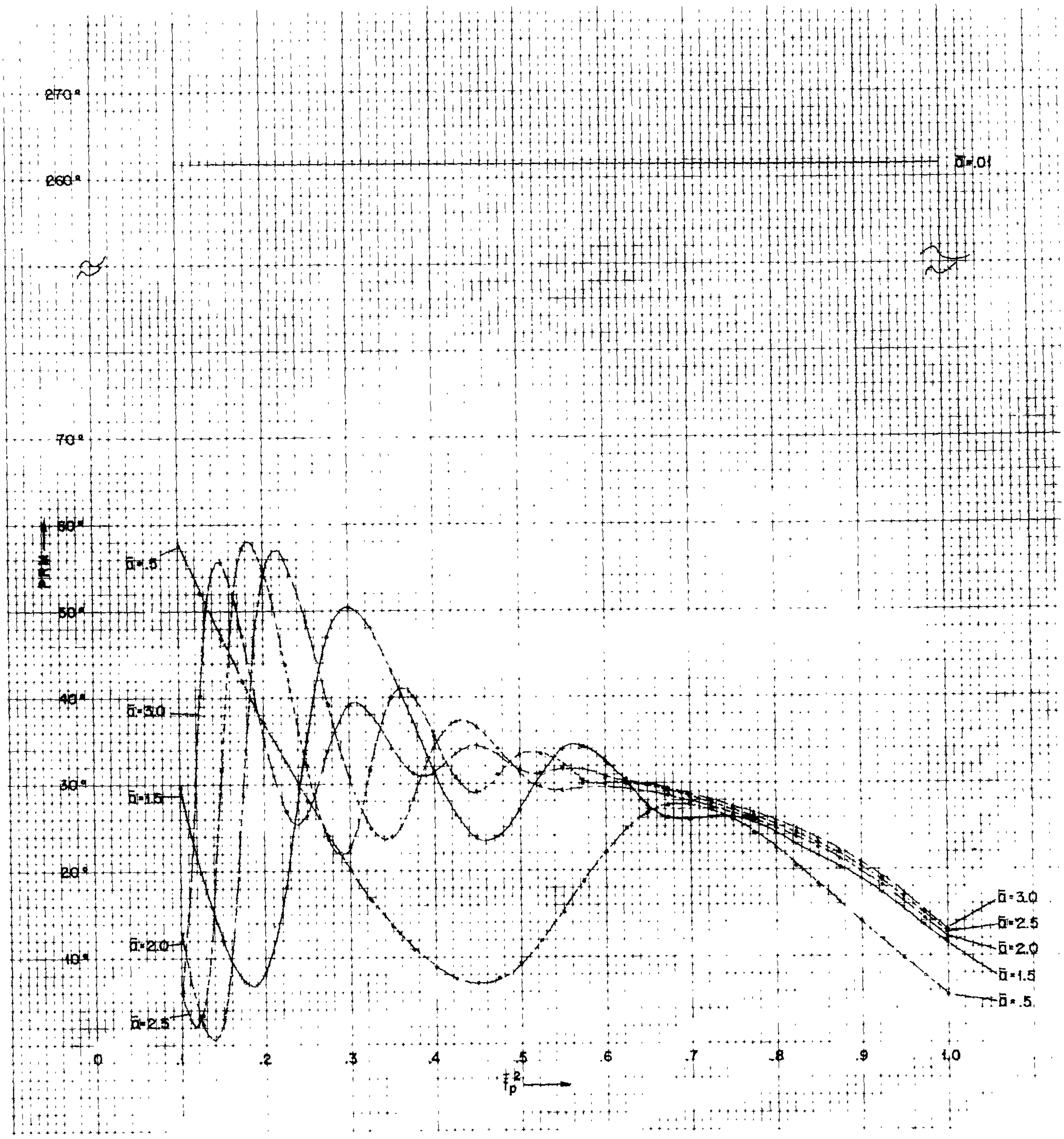

270°
260°
70°
60°
50°
40°
30°
20°
10°
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
$\bar{a}$=.01
$\bar{a}$=.5
$\bar{a}$=3.0
$\bar{a}$=1.5
$\bar{a}$=2.0
$\bar{a}$=2.5
$\bar{a}$=3.0
$\bar{a}$=2.5
$\bar{a}$=2.0
$\bar{a}$=1.5
$\bar{a}$=.5
f_p^2

Figure 62

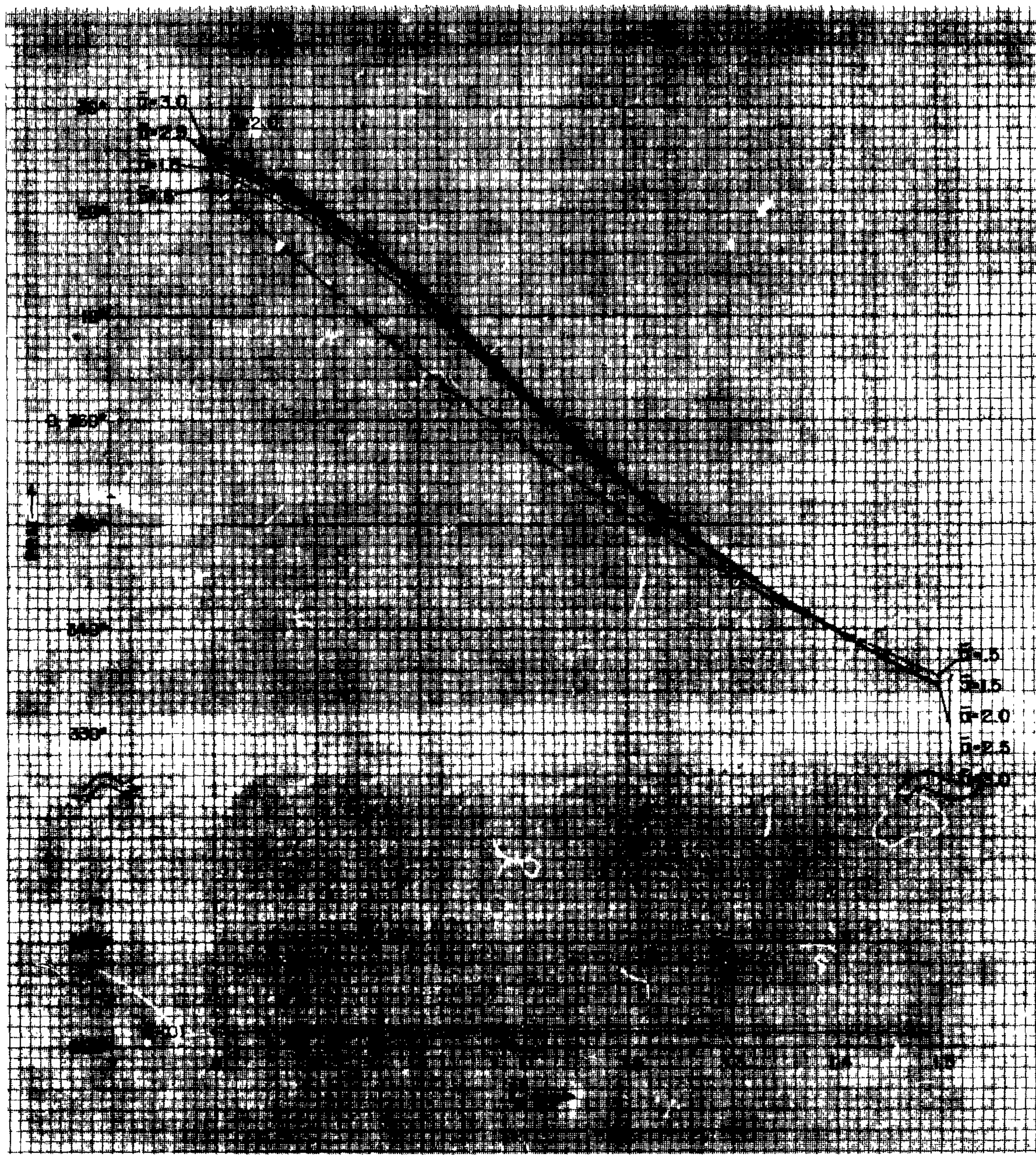
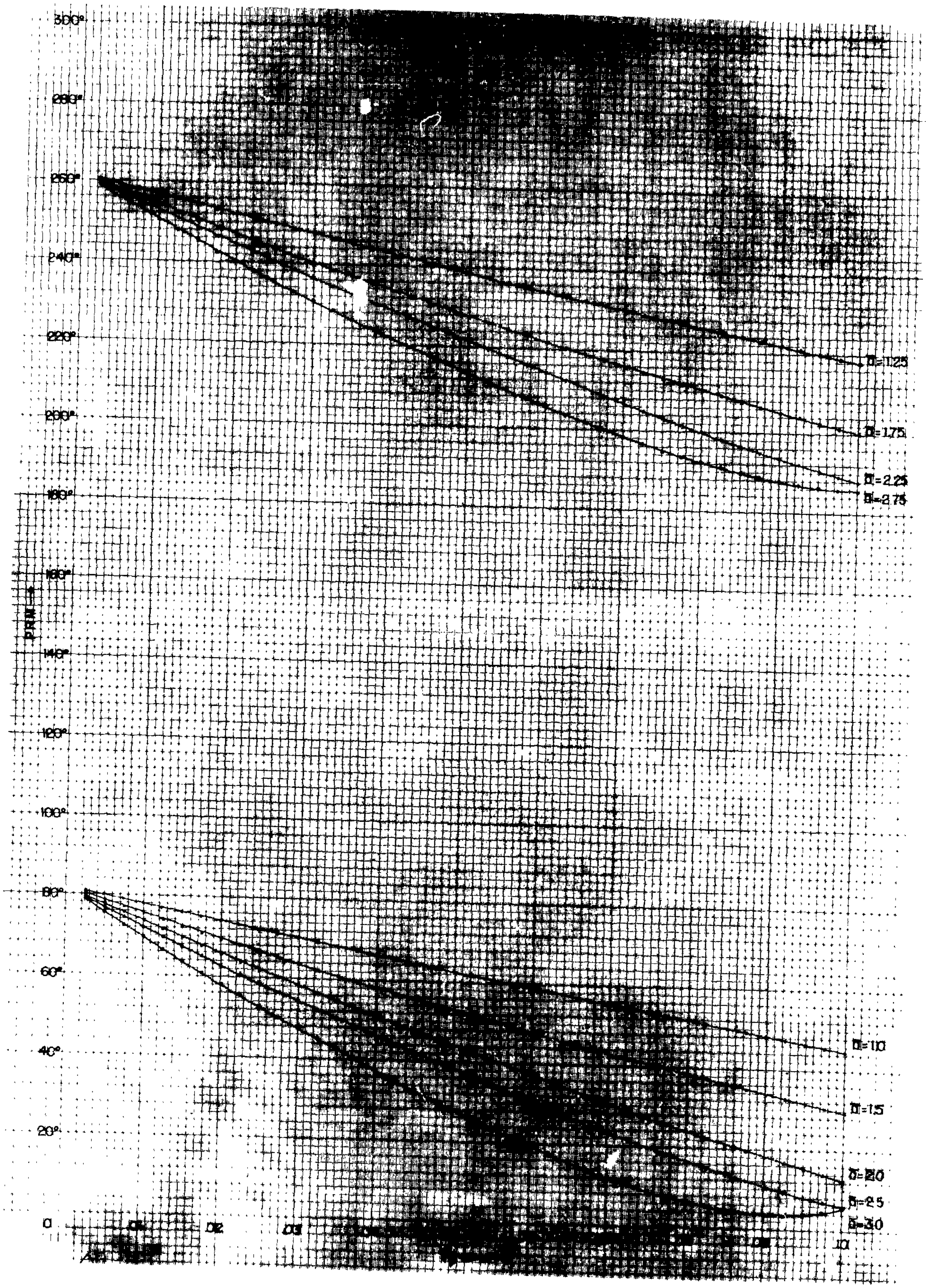


Figure 63



300°
280°
260°
240°
220°
200°
180°
160°
140°
120°
100°
80°
60°
40°
20°
0
PRN
$\bar{n}$=1.25
$\bar{n}$=1.75
$\bar{n}$=2.25
$\bar{n}$=2.75
$\bar{n}$=1.0
$\bar{n}$=1.5
$\bar{n}$=2.0
$\bar{n}$=2.5
$\bar{n}$=3.0
0.2
0.3
1.0

Figure 64

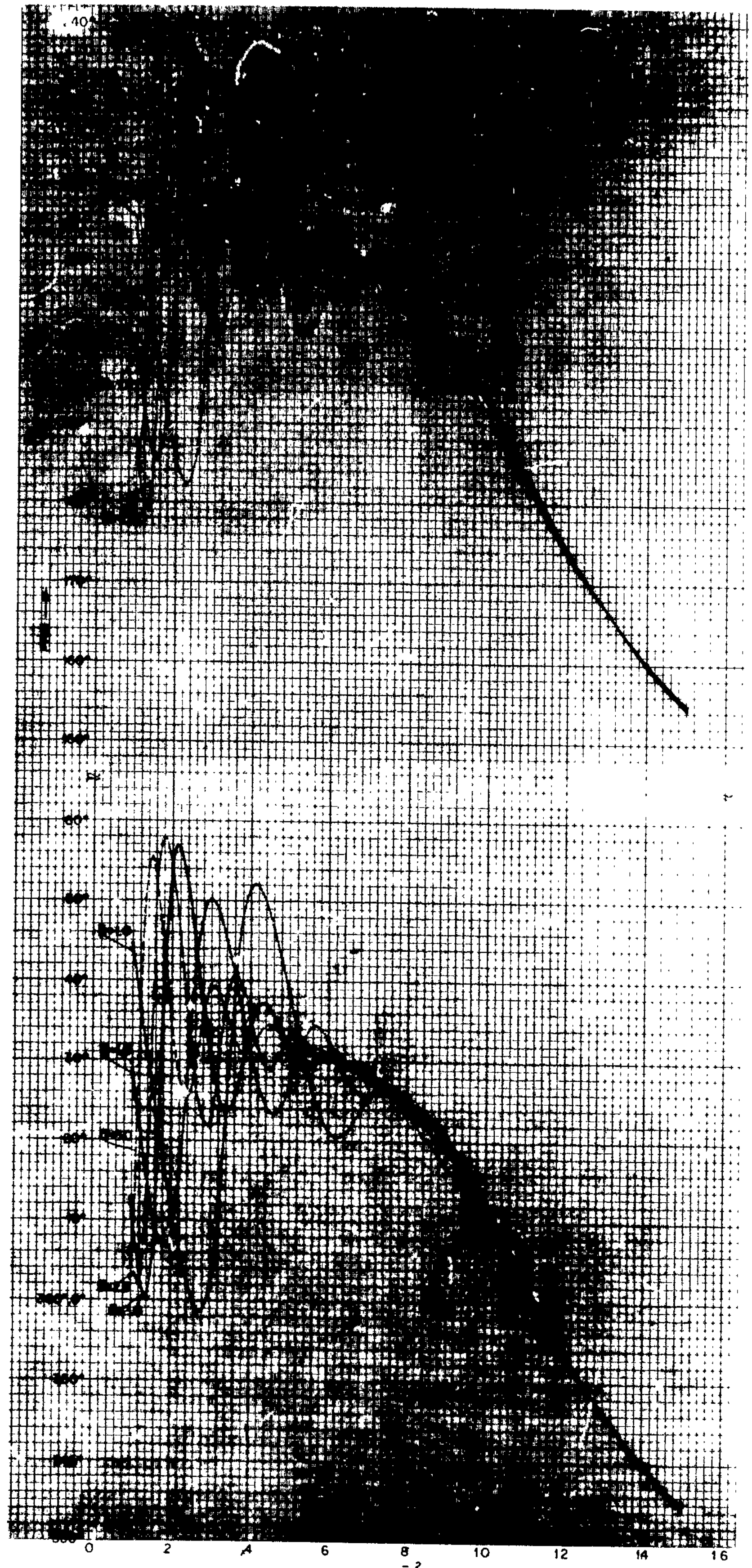


Figure 65

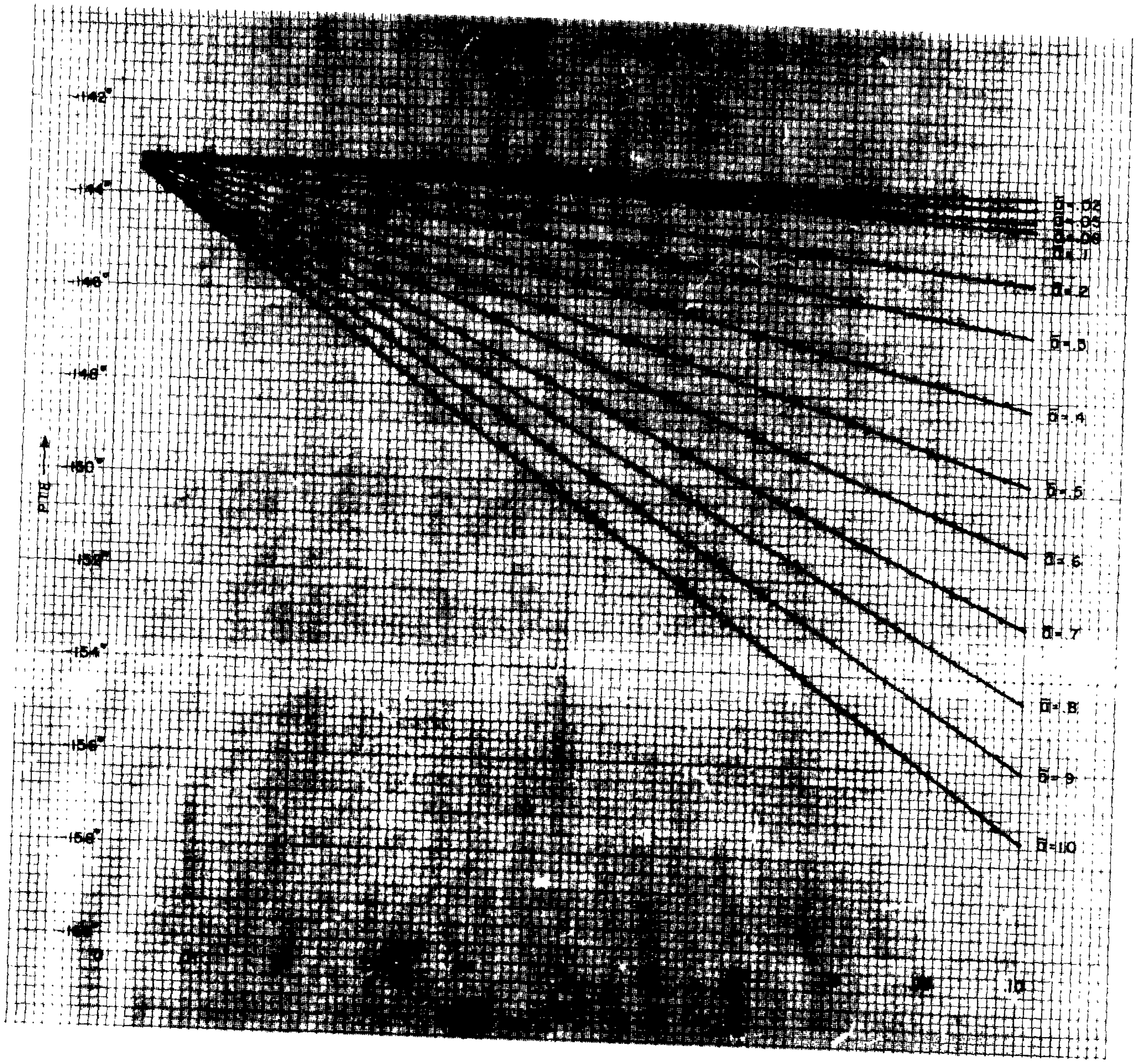
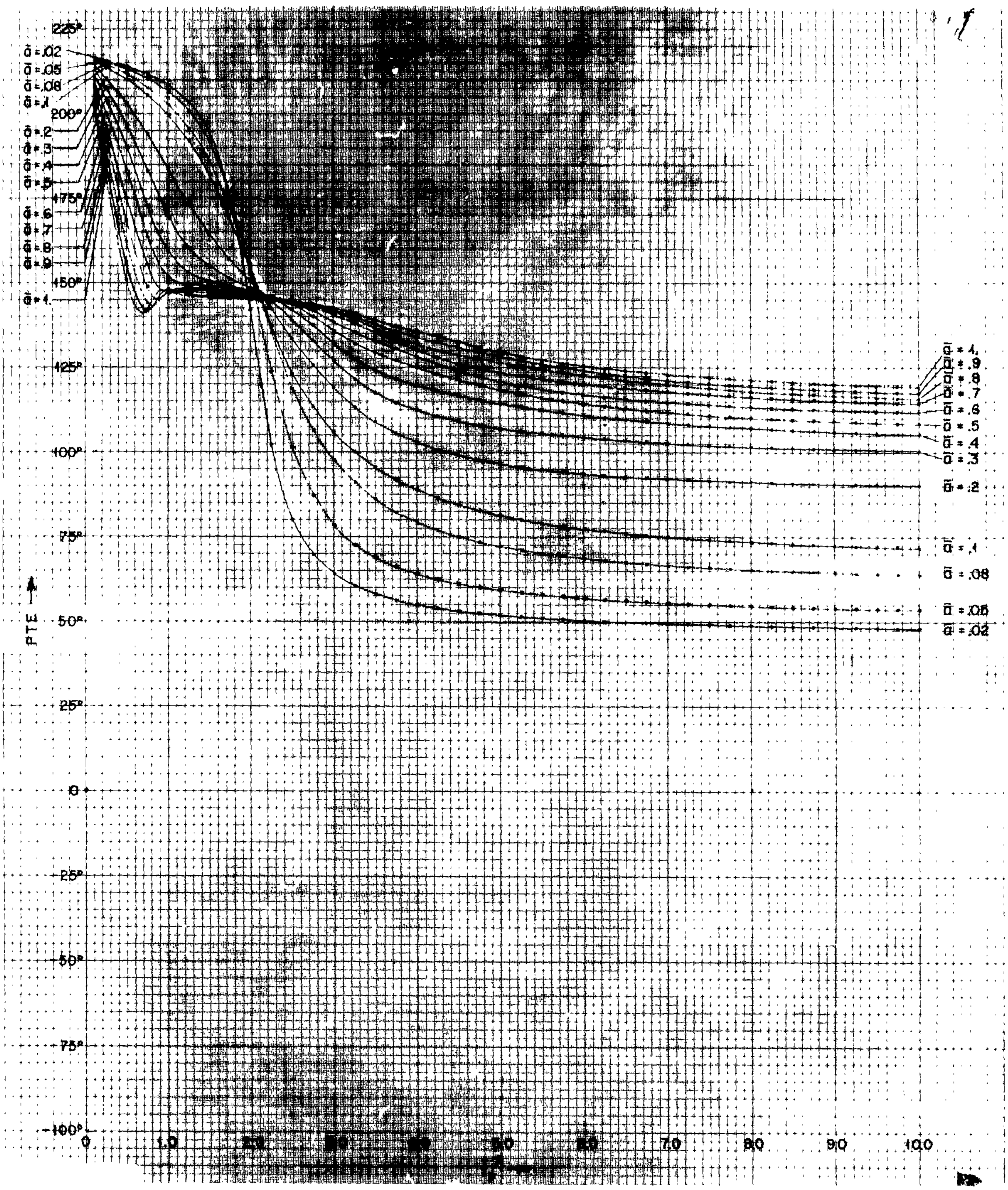
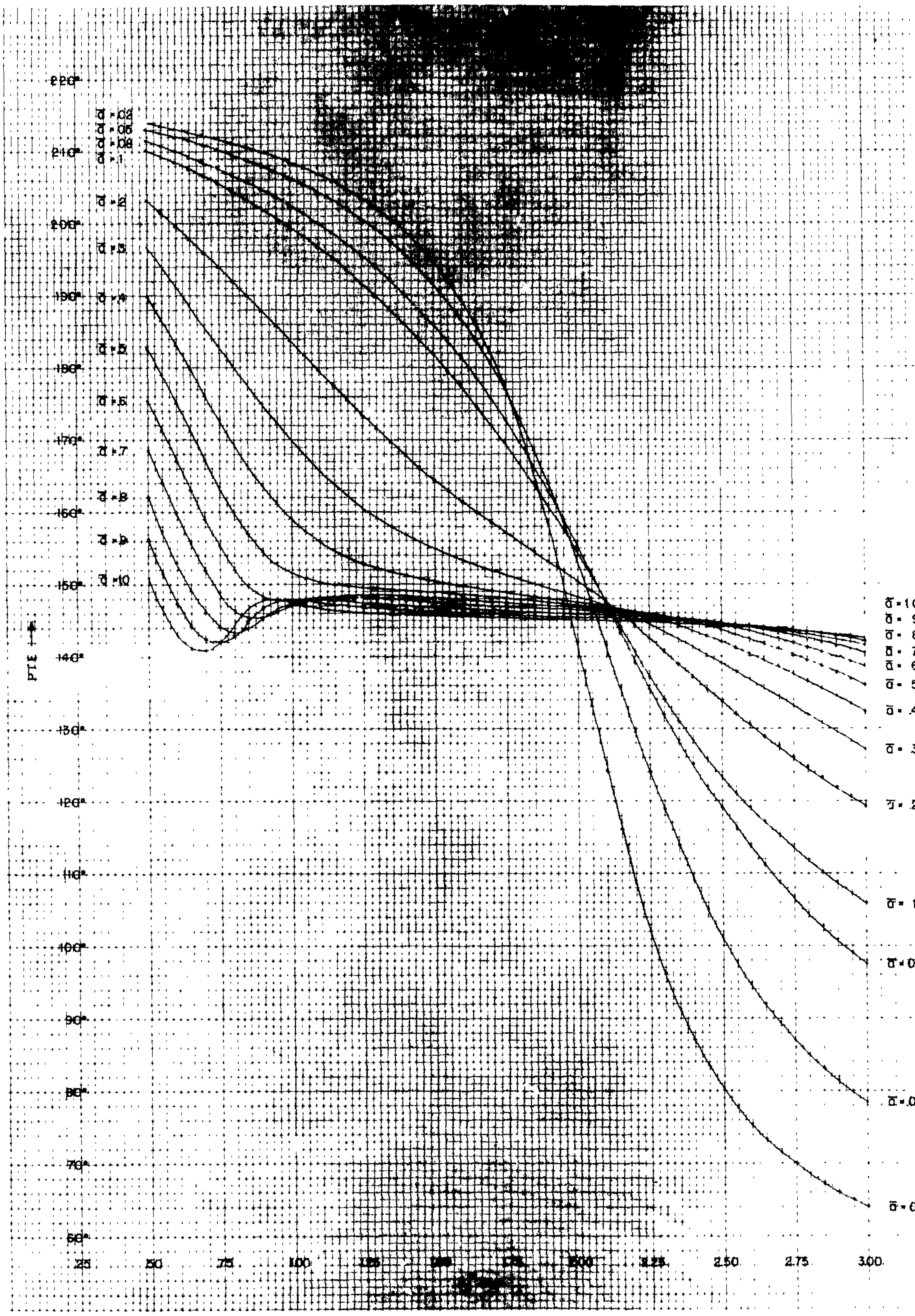


Figure 66



225°
200°
175°
150°
125°
100°
75°
50°
25°
0
-25°
-50°
-75°
-100°
PTE
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
ā = .02
ā = .05
ā = .08
ā = .1
ā = .2
ā = .3
ā = .4
ā = .5
ā = .6
ā = .7
ā = .8
ā = .9
ā = 1.

Figure 61



220°
210°
200°
190°
180°
170°
160°
150°
140°
130°
120°
110°
100°
90°
80°
70°
60°
PTE
2.50
2.75
3.00
σ̄ = 10
σ̄ = 9
σ̄ = 8
σ̄ = 7
σ̄ = 6
σ̄ = 5
σ̄ = .4
σ̄ = .3
σ̄ = .2
σ̄ = .1
σ̄ = .08
σ̄ = .05
σ̄ = .02

Figure 68

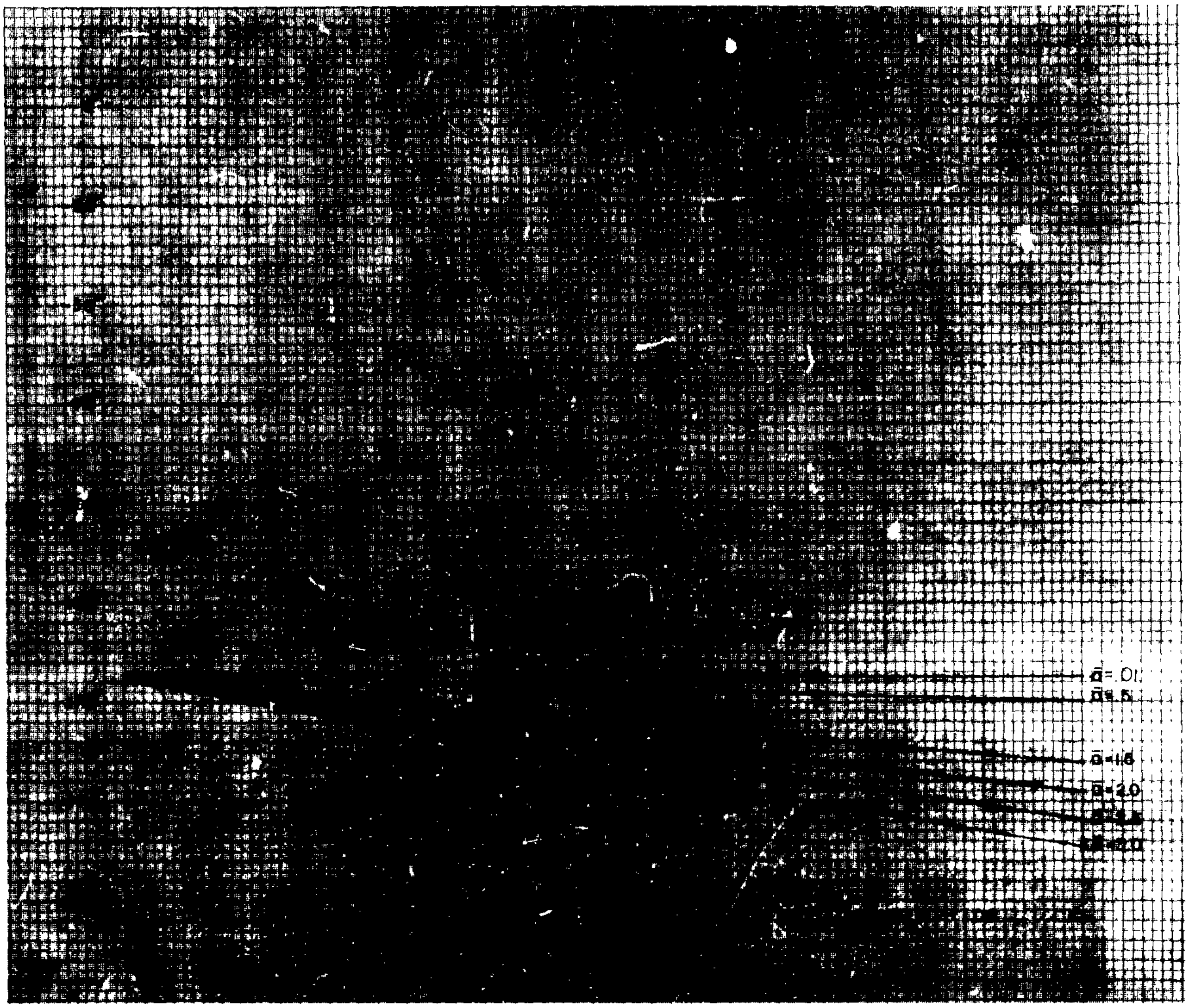


Figure 69

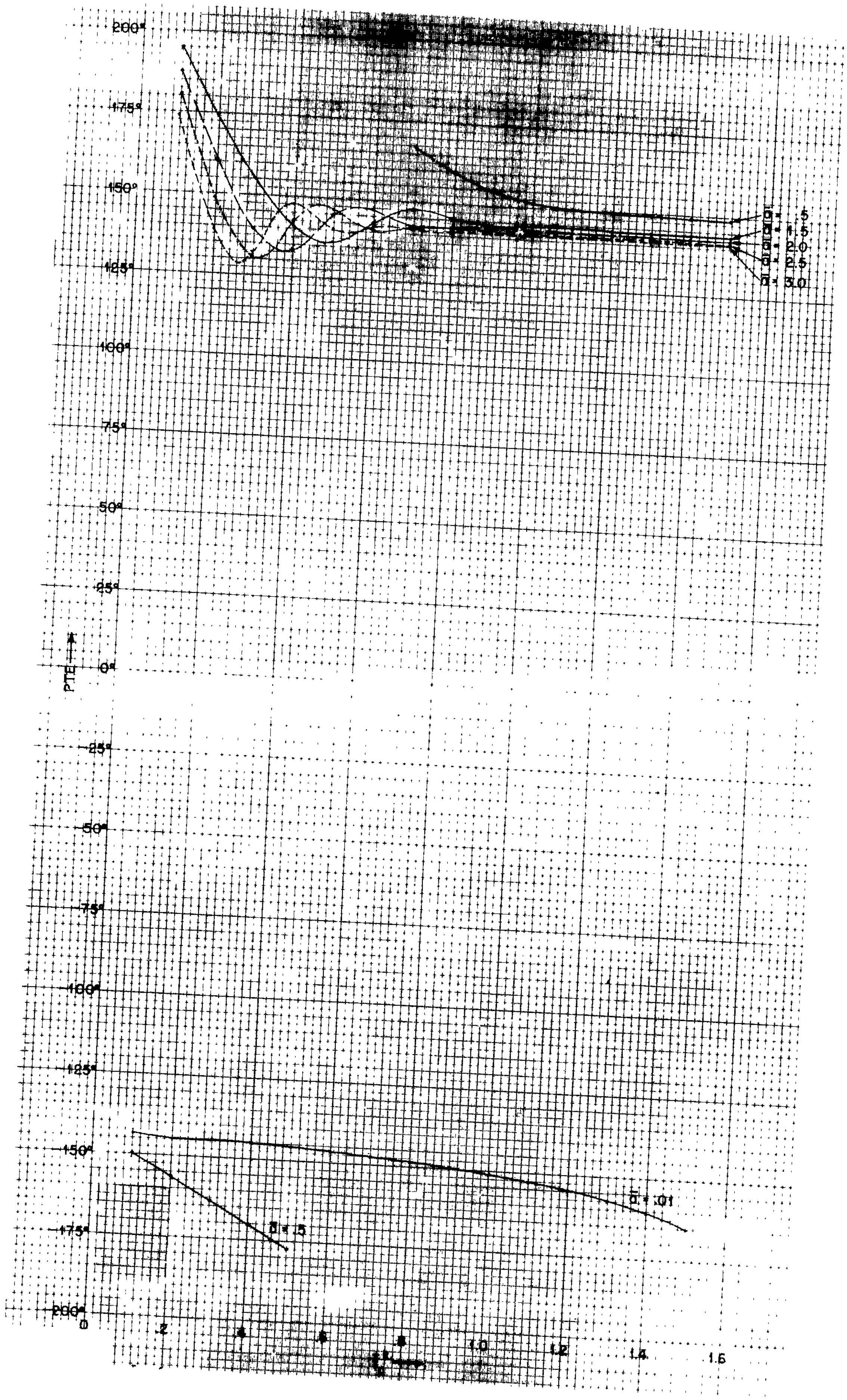
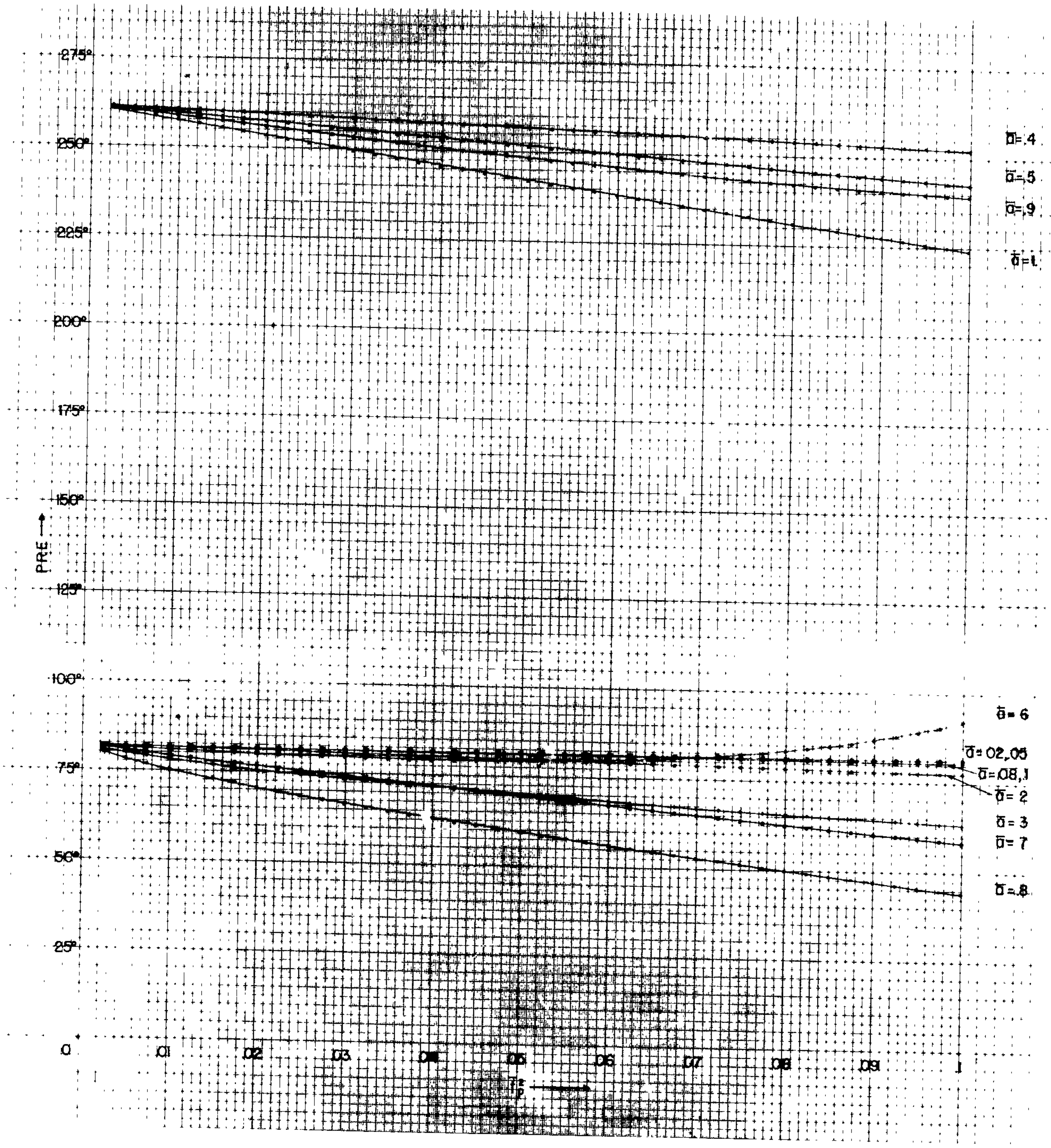

200°
175°
150°
125°
100°
75°
50°
25°
0°
PTE
-25°
-50°
-75°
-100°
-125°
-150°
-175°
-200°
$\bar{d}$ = .5
$\bar{d}$ = 1.5
$\bar{d}$ = 2.0
$\bar{d}$ = 2.5
$\bar{d}$ = 3.0
$\bar{d}$ = .01
$\bar{d}$ = .5
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6

Figure 10



275°
250°
225°
200°
175°
150°
125°
100°
75°
50°
25°
0
PRE
.01
.02
.03
.04
.05
.06
.07
.08
.09
.1
σ̄ =.4
σ̄ =.5
σ̄ =.9
σ̄ =1
σ̄ = 6
σ̄ = 02,05
σ̄ =.08,.1
σ̄ = 2
σ̄ = 3
σ̄ = 7
σ̄ =.8

Figure /1

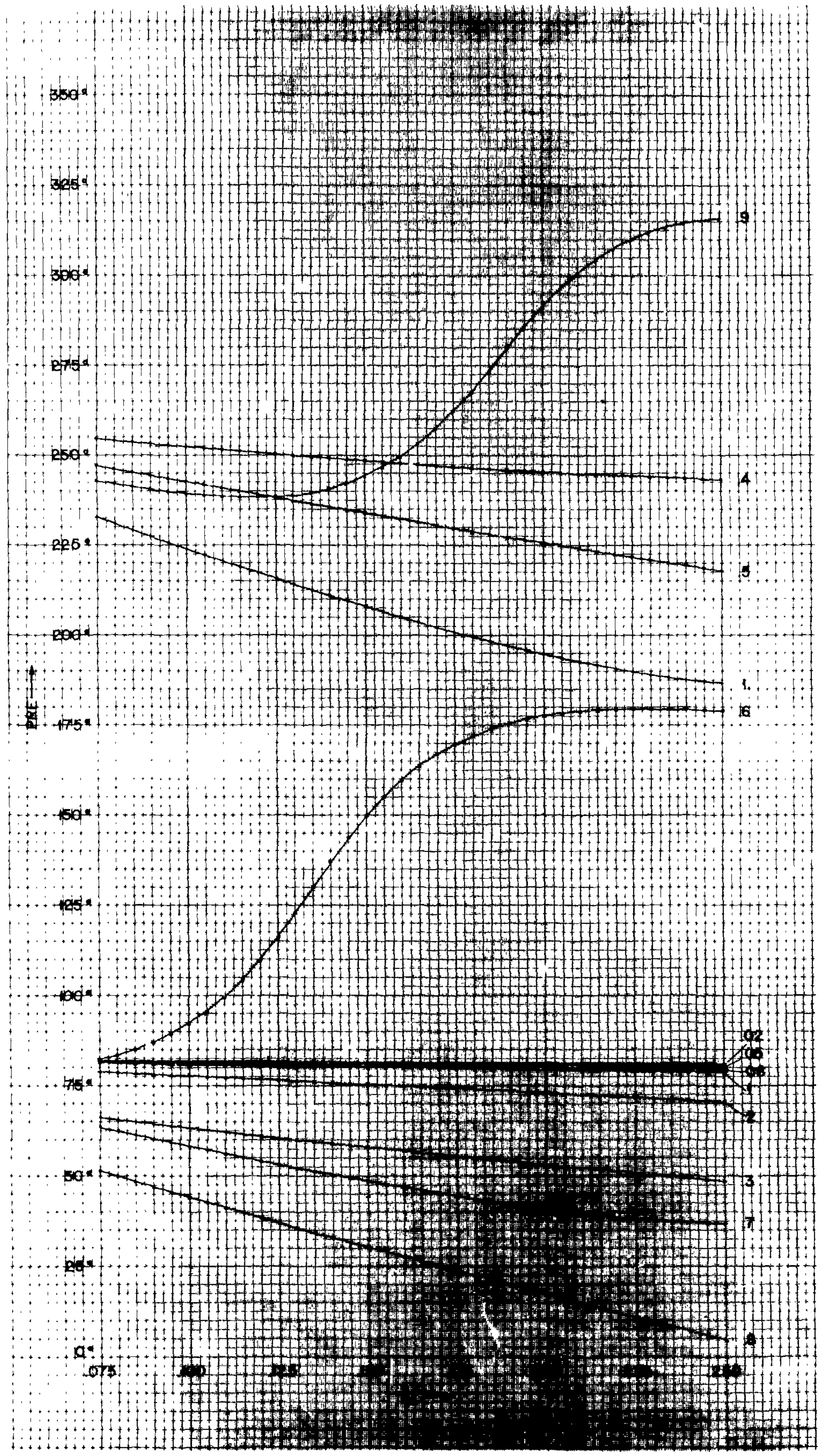
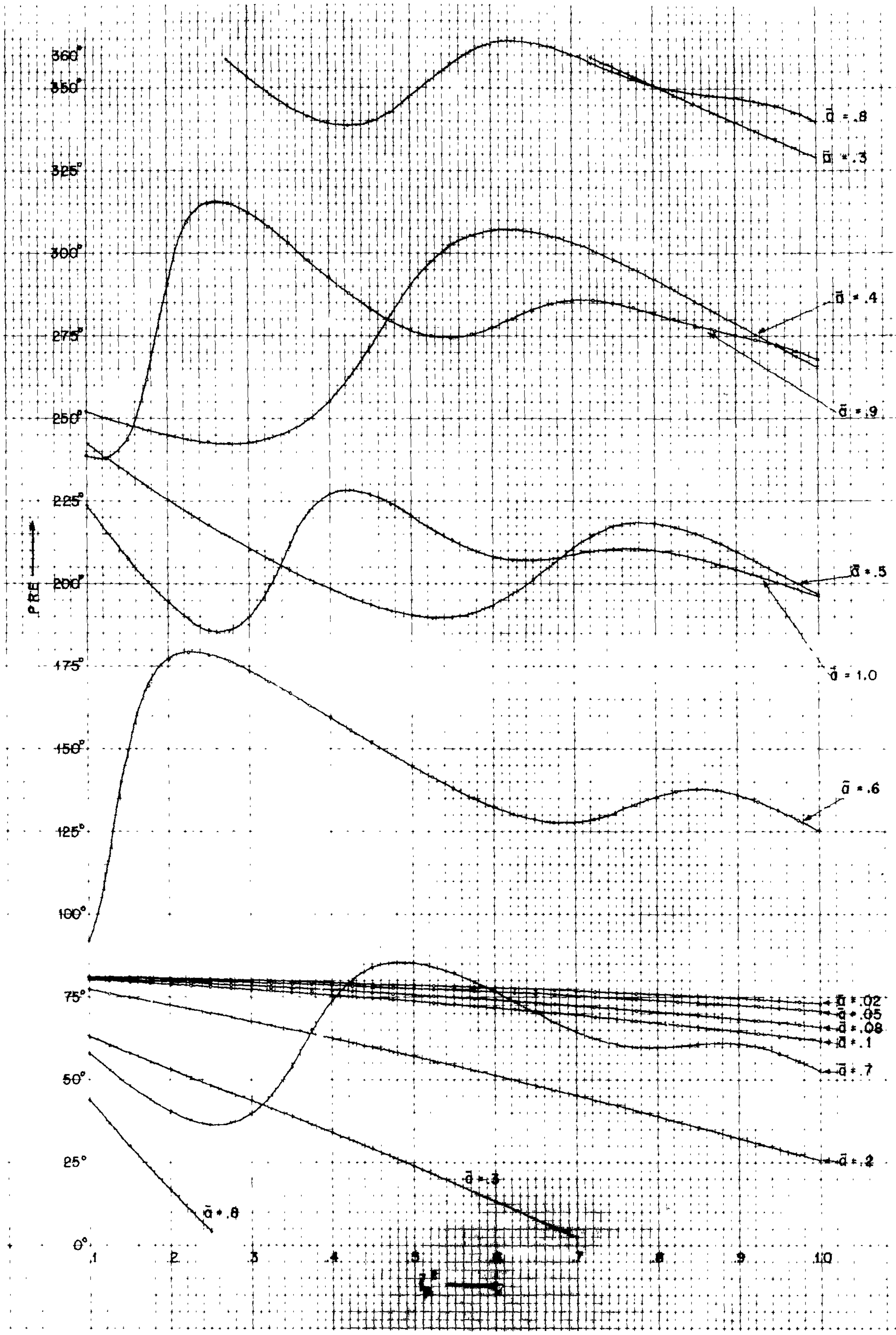

350°
325°
300°
275°
250°
225°
200°
175°
150°
125°
100°
75°
50°
25°
0°
.075
9
4
5
1
6
3
7
8

Figure /2



360°
350°
325°
300°
275°
250°
225°
200°
PRE
175°
150°
125°
100°
75°
50°
25°
0°
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
ā = .8
ā = .3
ā = .4
ā = .9
ā = .5
ā = 1.0
ā = .6
ā = .02
ā = .05
ā = .08
ā = .1
ā = .7
ā = .2
ā = .3
ā = .8

Figure 73

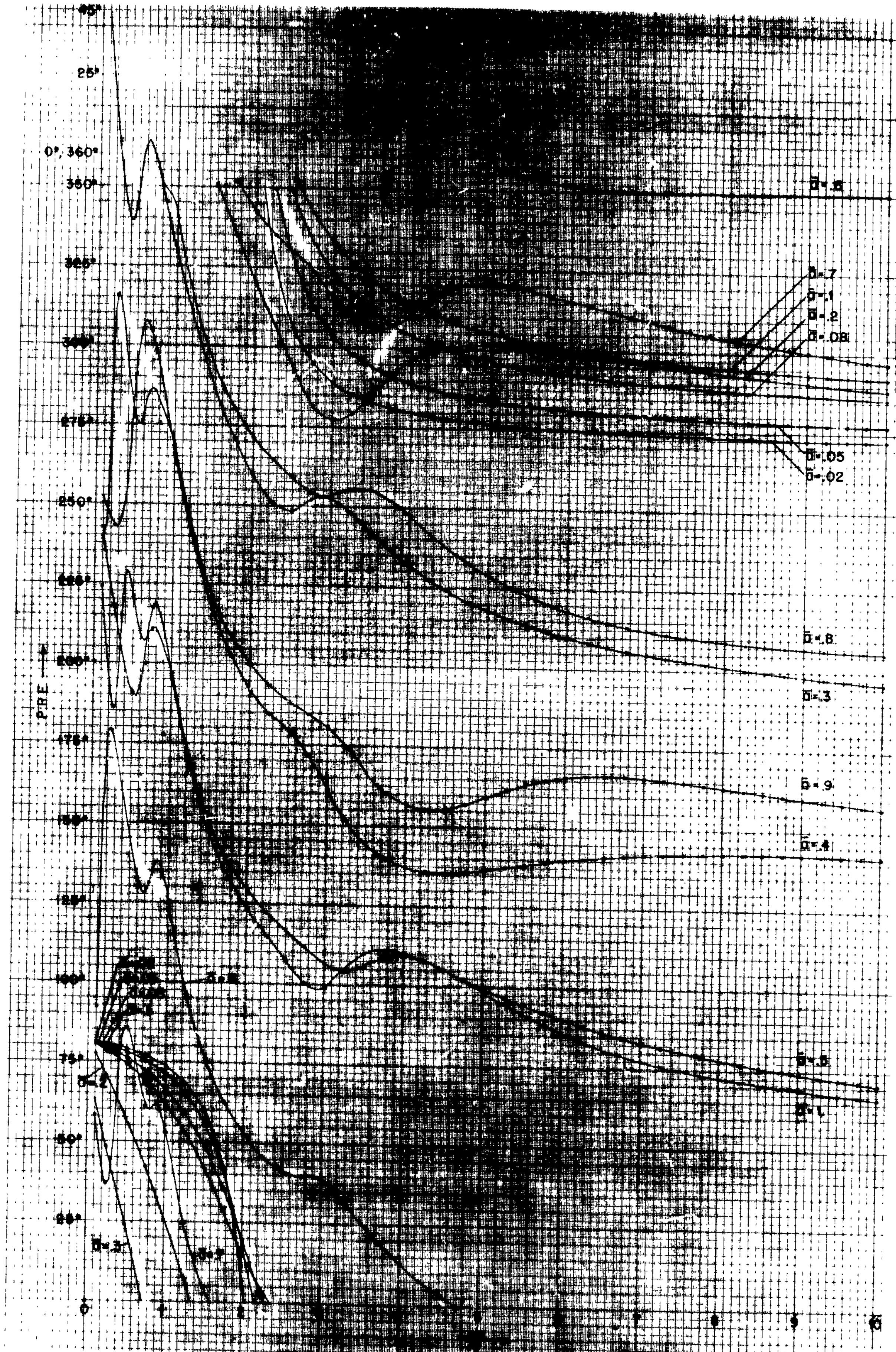


Figure 4

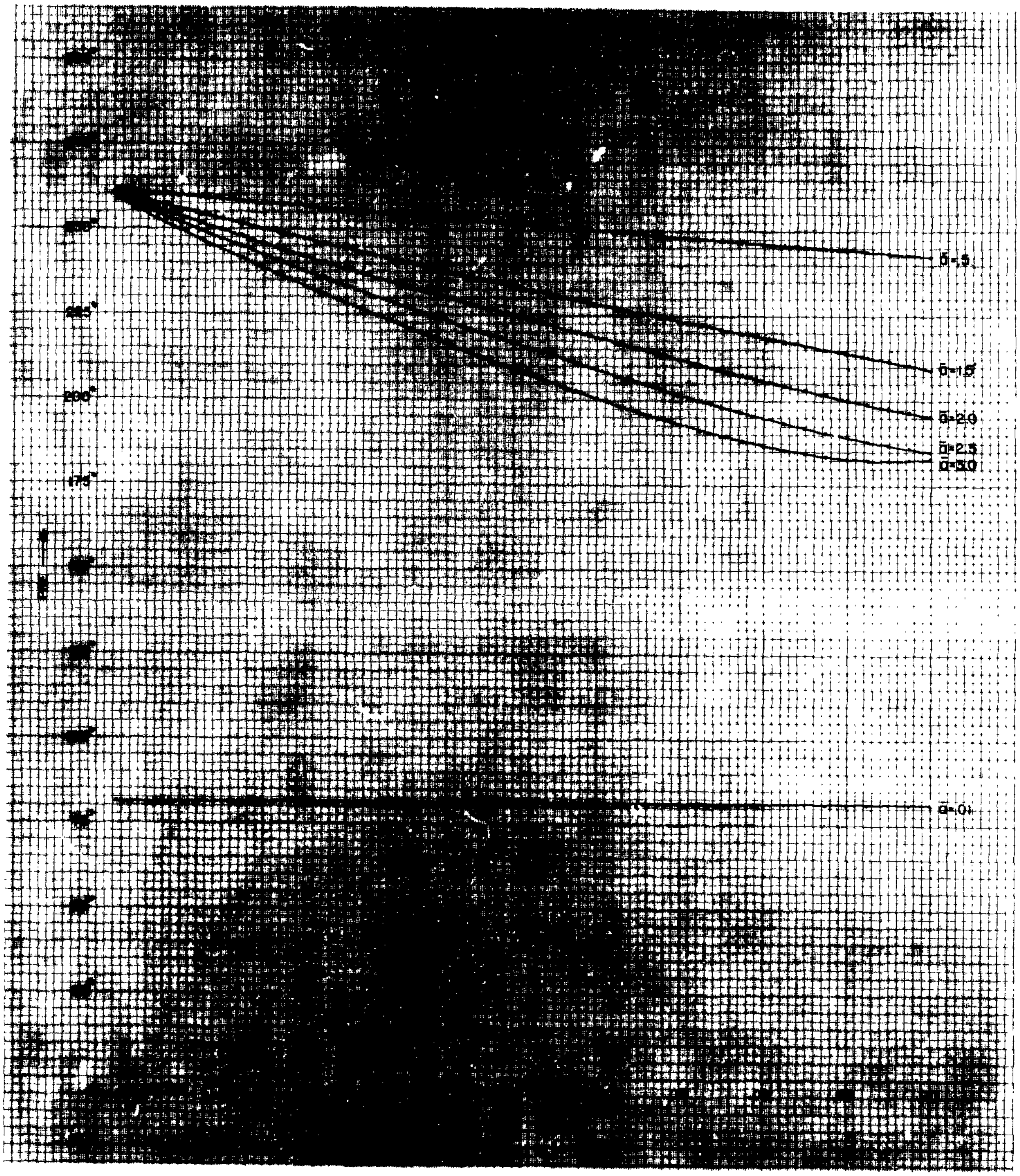
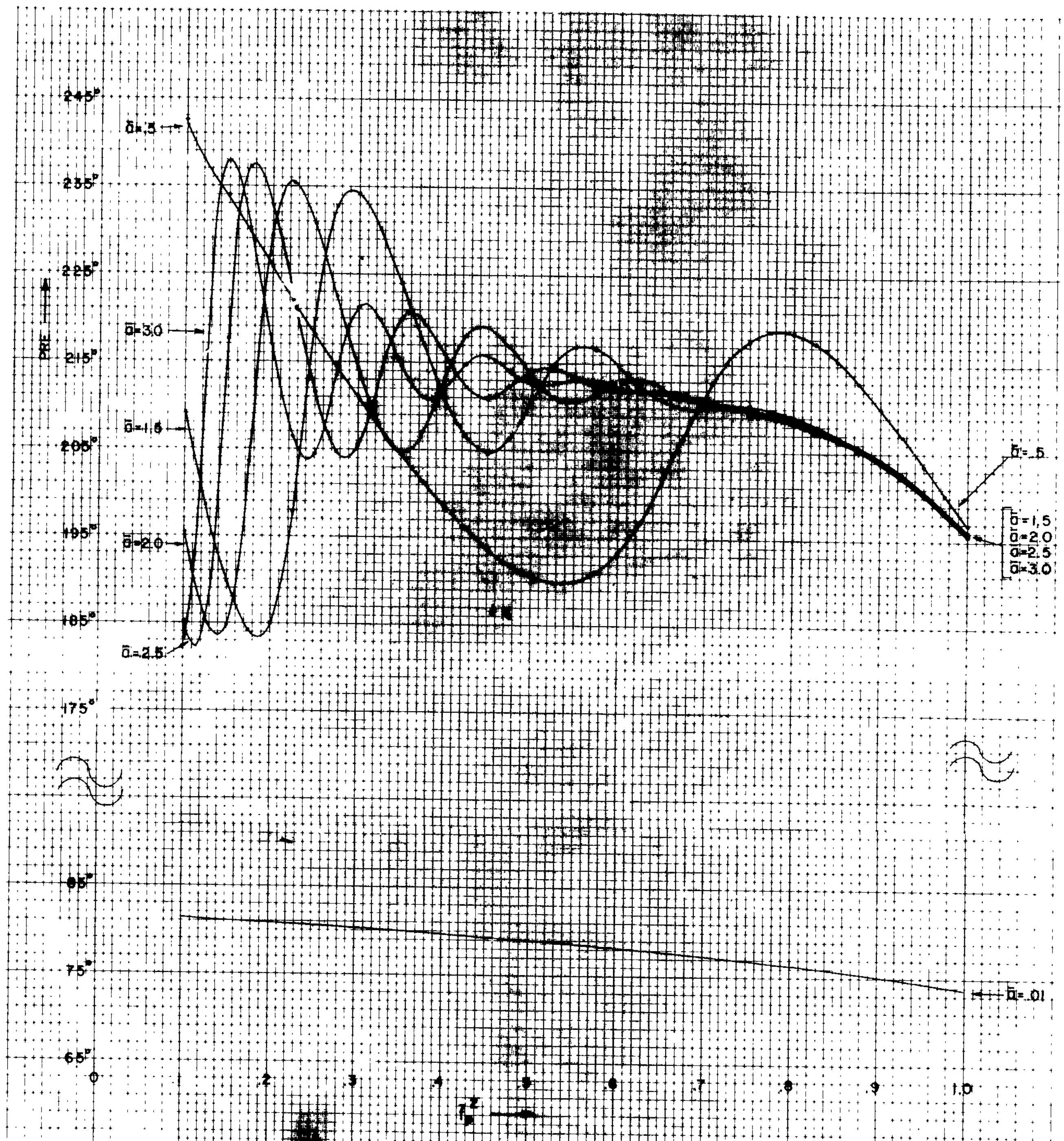
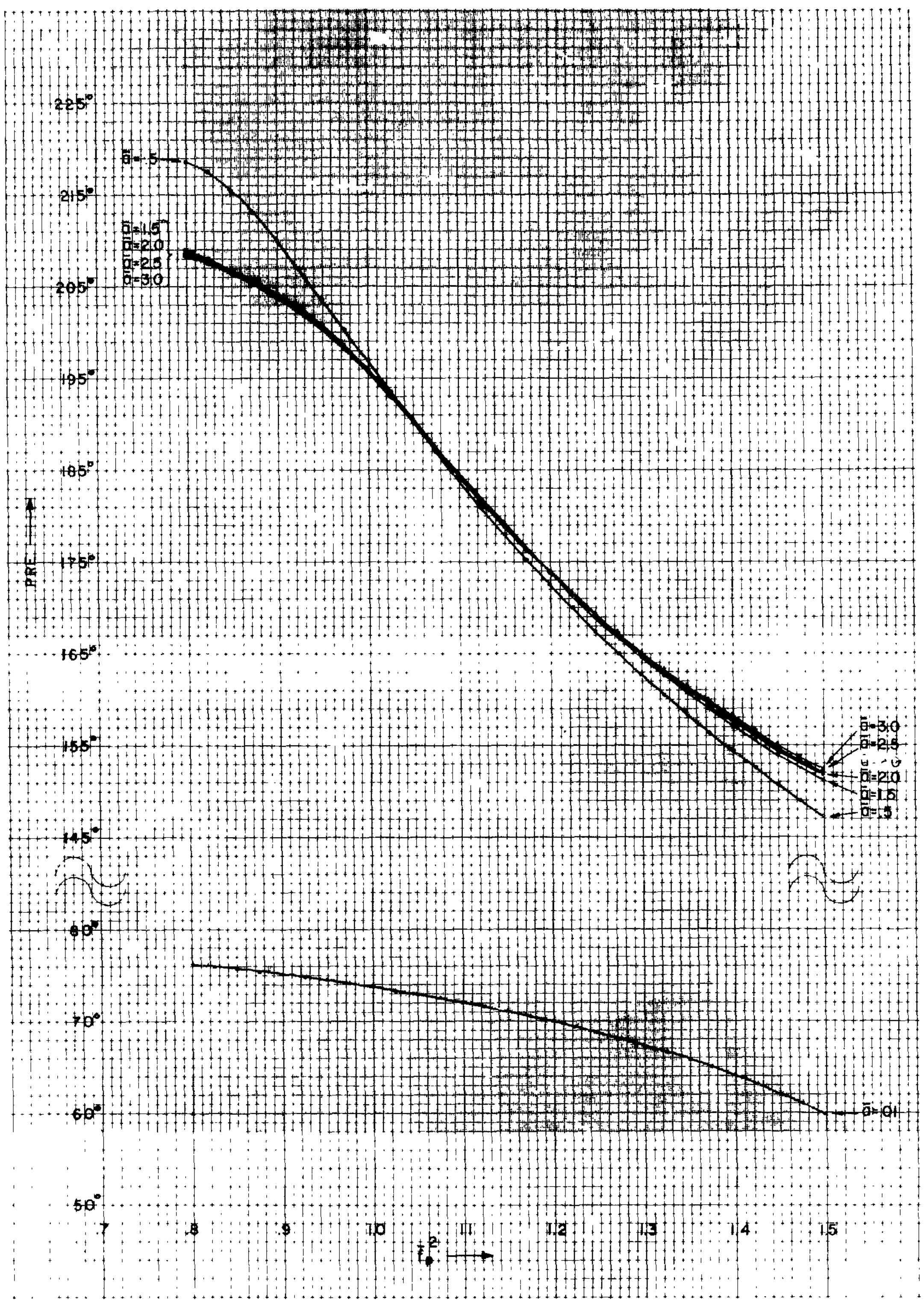


Figure 75



245°
235°
225°
215°
205°
195°
185°
175°
85°
75°
65°
DRE
ā=.5
ā=3.0
ā=1.5
ā=2.0
ā=2.5
ā=.5
ā=1.5
ā=2.0
ā=2.5
ā=3.0
ā=.01
0
.2
.3
.4
.5
.6
.7
.8
.9
1.0

Figure 76



225°
215°
205°
195°
185°
175°
165°
155°
145°
80°
70°
60°
50°
PRE
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
$\bar{a}$=.5
$\bar{a}$=1.5
$\bar{a}$=2.0
$\bar{a}$=2.5
$\bar{a}$=3.0
$\bar{a}$=.01

Figure 77

APPENDIX II

The PRM and PRE curves possess a few interesting characteristics which will be briefly discussed here. It appears that for very low electron densities, i.e. for very small $\bar{f}_p^2$, the phase of the reflection coefficient for both the transverse magnetic case and the transverse electric case becomes independent of the electron density. Given a normalized collision frequency $\bar{\nu}$ and any normalized radius $\bar{a}$ PRM always takes one of two values which differ by π, and the same is true for PRE.

Why this is so can easily be seen from the equations arising from the solution of the boundary value problem. Assuming $\bar{f}_p^2$ to be small we may write

$$\epsilon' = 1 - \frac{\bar{f}_p^2 (1-i\bar{\nu})}{1+\bar{\nu}^2} = 1-\delta$$

and

$$\sqrt{\epsilon'} = 1 - \frac{\delta}{2}$$

where

$$\delta = \frac{\bar{f}_p^2 (1-i\bar{\nu})}{1+\bar{\nu}^2}$$

Discarding all terms involving powers of δ higher then the 2nd we obtain

$$\text{PRM} \doteq \text{PRE} + \pi = \frac{\pi}{2} - \tan^{-1} \bar{\nu} + \tan^{-1} \left[\frac{\bar{f}_p^2 K (2\pi\bar{a})}{-J_1(4\pi a)}\right]$$

where

$$K(2\pi\bar{a}) = 2\pi^4\bar{a}^3 \sum_{n=-\infty}^{\infty} (-1)^n \left[J_{n-1}(2\pi\bar{a})\, J_{n+1}(2\pi\bar{a}) - J_n^{\,2}(2\pi\bar{a})\right]^2$$

(PRM $\neq$ PRE but PRM = PRE + π because we have defined the reflection coefficient in terms of the scattered electric field in the transverse magnetic case and in terms of the scattered magnetic field in the transverse electric case.)

This expression shows at once that for small $\bar{f}_p^{\,2}$

PRM = PRE + π = $-\frac{\pi}{2} - \tan^{-1}\bar{\nu}$ when $\bar{a}$ is such that $J_1(4\pi\bar{a}) > 0$

PRM = PRE + π = $\frac{\pi}{2} - \tan^{-1}\bar{\nu}$ when $\bar{a}$ is such that $J_1(4\pi\bar{a}) < 0$

PRM = PRE + π = $-\tan^{-1}\bar{\nu}$ when $\bar{a}$ is such that $J_1(4\pi\bar{a}) = 0$ and $K(2\pi\bar{a}) < 0$

PRM = PRE + π = $\pi - \tan^{-1}\bar{\nu}$ when $\bar{a}$ is such that $J_1(4\pi\bar{a}) = 0$ and $K(2\pi\bar{a}) > 0$

also

$$\frac{d\ \text{PRM}}{d\ \bar{f}_p^{\,2}} = \frac{d\ \text{PRE}}{d\ \bar{f}_p^{\,2}} = \frac{\left[\frac{K(2\pi\bar{a})}{-J_1(4\pi\bar{a})}\right]}{1 + \left[\bar{f}_p^{\,2}\,\frac{K(2\pi\bar{a})}{-J_1(4\pi\bar{a})}\right]^2} \simeq \frac{K(2\pi\bar{a})}{-J_1(4\pi\bar{a})}$$

Hence for small $\bar{f}_p^{\,2}$ the slope of a curve representing PRM or PRE as a function of $\bar{f}_p^{\,2}$ depends for a given $\bar{a}$ on both $J_1(4\pi\bar{a})$ and $K(2\pi\bar{a})$. Suppose $\bar{a}$ is varied by small increments until $J_1(4\pi\bar{a})$ changes sign. Since $K(2\pi\bar{a})$ will not change sign at the same time as $J_1(4\pi\bar{a})$, PRM or PRE will change by π, and the slope of a curve representing PRM or PRE as a function of $\bar{f}_p^{\,2}$ will change sign. If $\bar{a}$ is varied further $K(2\pi\bar{a})$ will eventually change sign too and the slope will change sign again. For $\bar{a}$ such that

$J_1(4\pi\bar{a}) = 0$ the slope of a curve representing PRM or PRE as a function of $\bar{f}_p^{\,2}$ will be $+\infty$ or $-\infty$ depending on whether $K(2\pi\bar{a})$ is positive or negative. Figure 78 shows in some detail the behavior of PRM for small $\bar{f}_p^{\,2}$ when $\bar{a}$ is near a value $\bar{a}_1$ where $J_1(4\pi\bar{a}_1) = 0$.

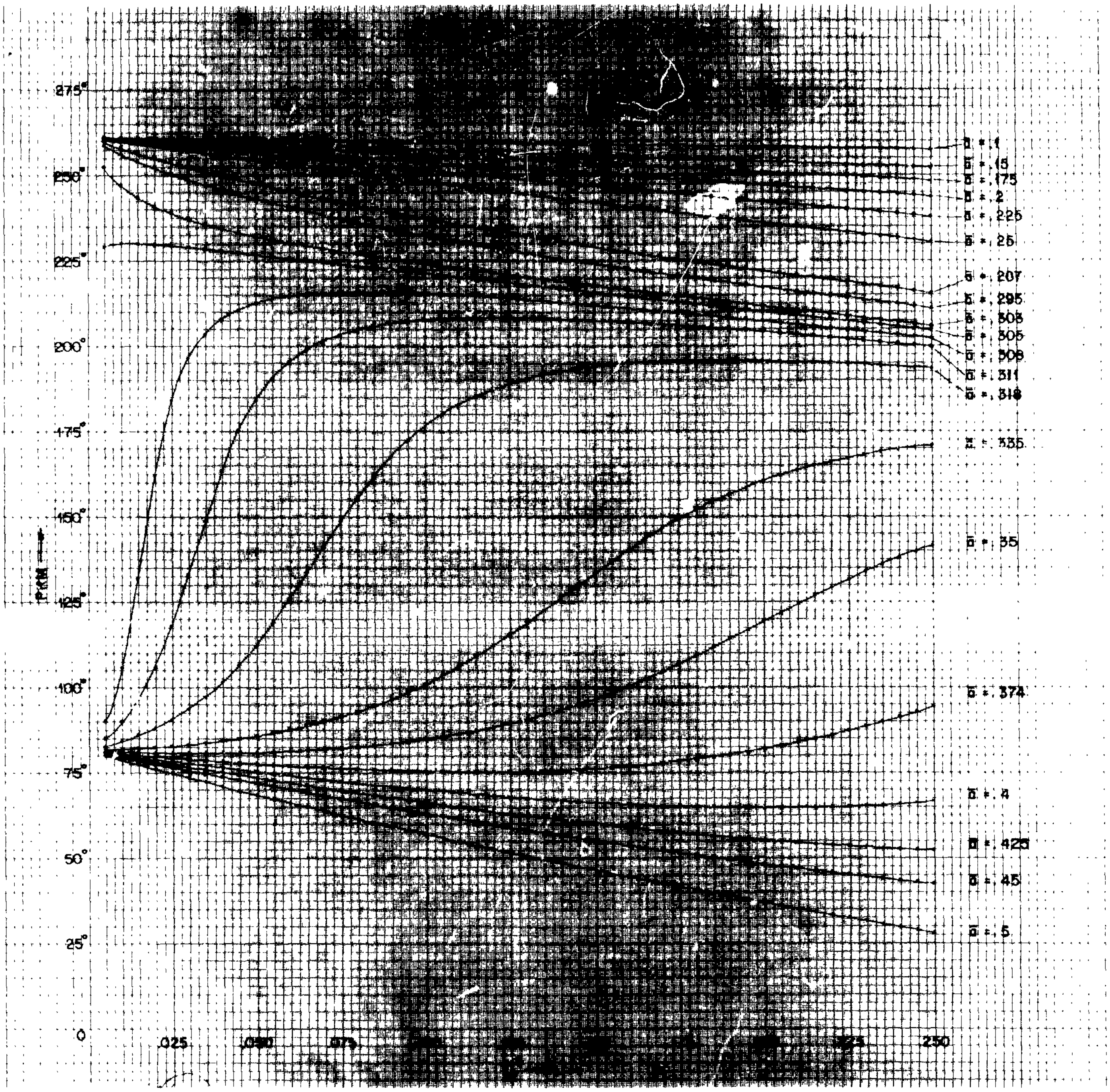


Figure 18